# THE
# PROCEEDINGS
# OF
# THE PHYSICAL SOCIETY

## Section B

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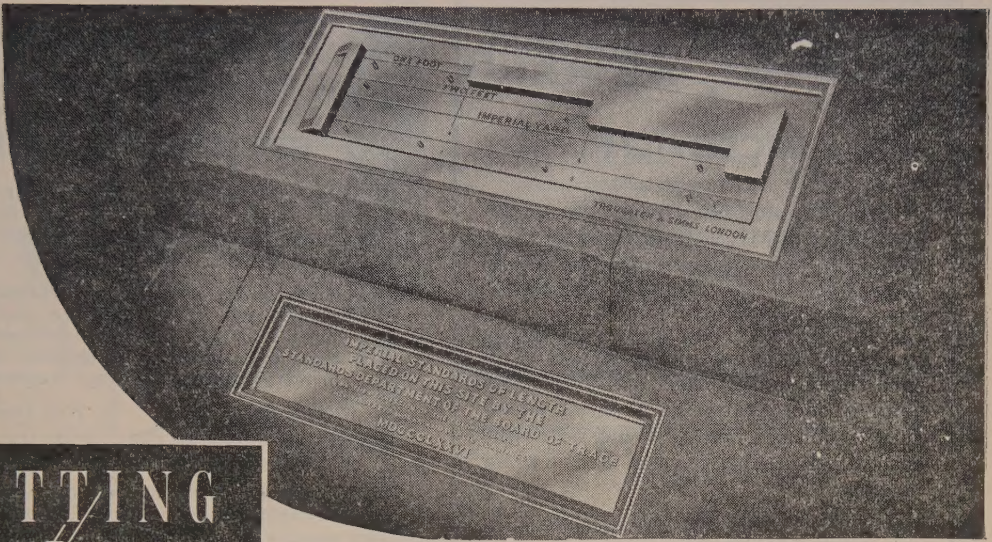
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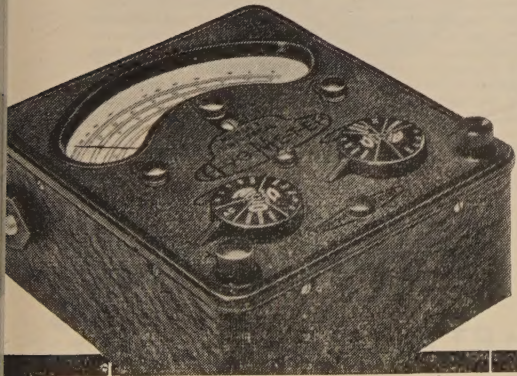

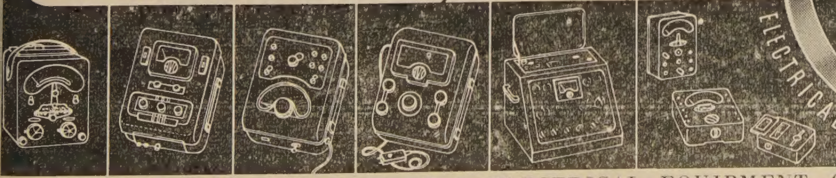

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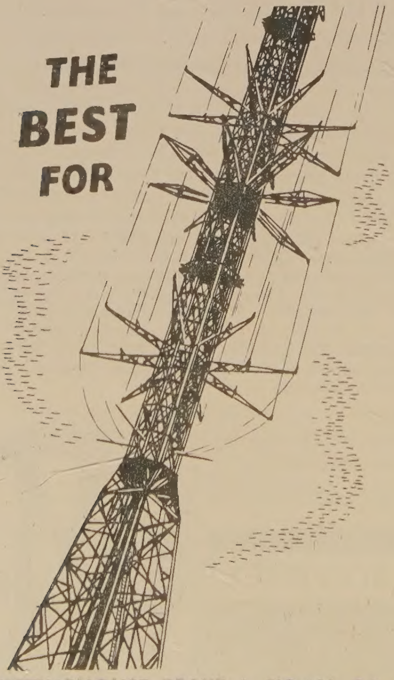

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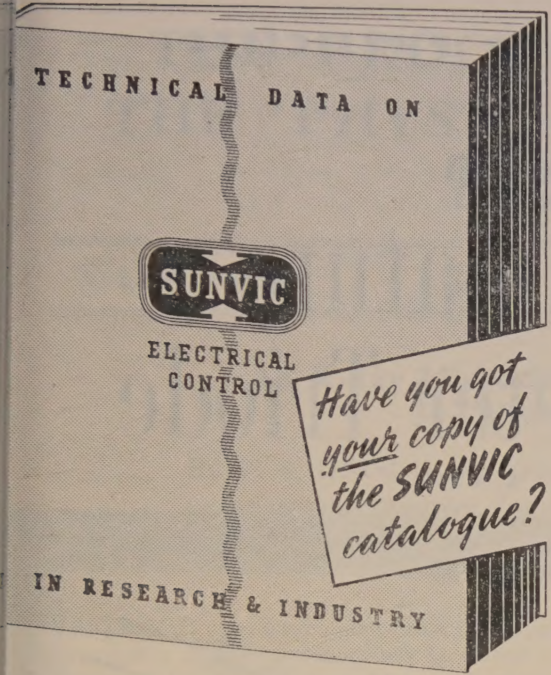

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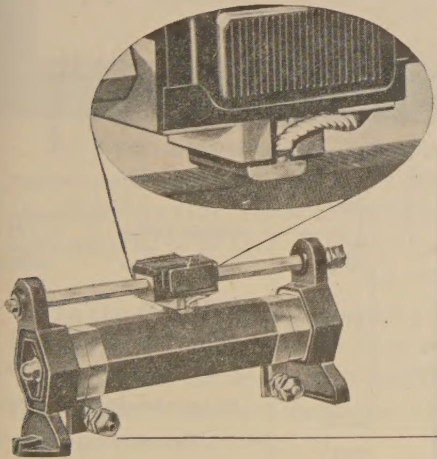

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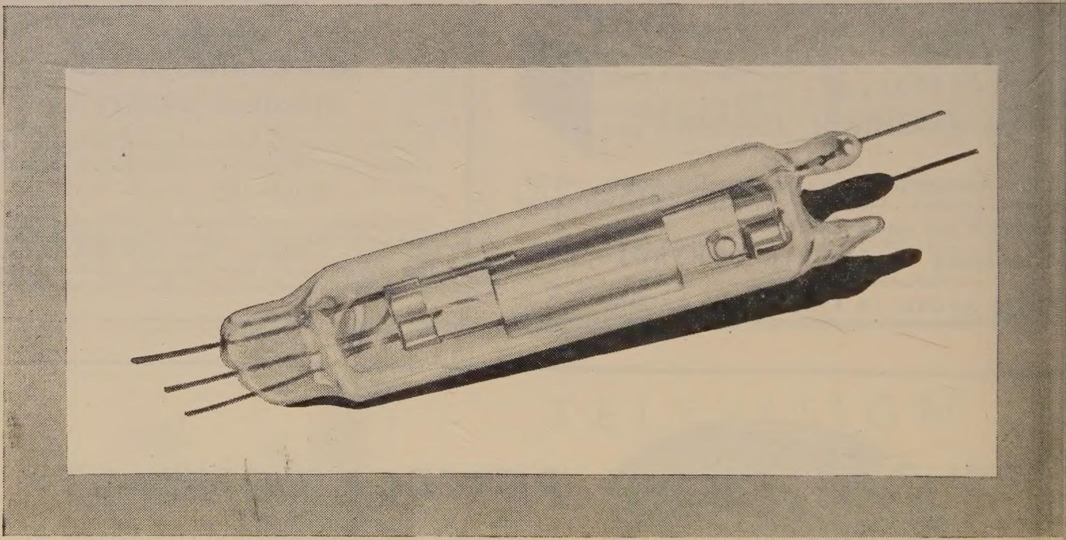

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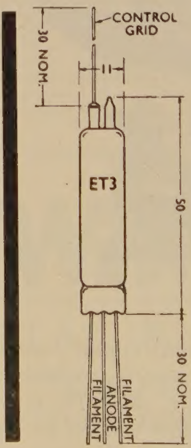

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Section B

Vol. 64, Part 5 1 May 1951 No. 377 B

# An X-Ray Micro-Beam Technique: I—Collimation

By P. B. HIRSCH and the late J. N. KELLAR *
Crystallographic Laboratory, Cavendish Laboratory, Cambridge

*Communicated by W. H. Taylor; MS. received* 29*th June* 1950, *and in amended form* 6*th November* 1950

*ABSTRACT.* An x-ray micro-beam technique is described which depends on the use of x-ray beams down to approximately 1 $\mu$ diameter, obtained by means of lead glass capillaries. The micro-beam precision camera incorporates collimator holders for aligning the capillaries, and enables simultaneous transmission and back-reflection photographs to be taken. The effect of the divergence of the x-ray beam on the photographs is considered, and a formula is derived for the mean value of the divergence at the specimen. The optimum collimating conditions are discussed in relation to maximum resolution and minimum exposure. The use of totally reflecting glass collimators is considered. An example is given of the application of the micro-beam technique to the study of cold-worked metals. It is found that, using a high intensity generator, single reflections can be obtained from crystals of diameter of the order of 2 $\mu$ in about 10 hours.

## § 1. INTRODUCTION

The recently developed micro-beam technique for the study of cold-worked metals (Kellar, Hirsch and Thorp 1950) required the use of high intensity x-ray beams a few microns in diameter. This paper deals with the micro-collimation of the x-ray beams; the high intensity x-ray generator is described by Gay, Hirsch, Thorp and Kellar (1951, Part II of the present paper, and referred to as II).

## § 2. COLLIMATORS

In the present technique lead glass capillaries or lead pinholes are used. The capillaries are preferred, as they can generally be used without a scatter stop. Table 1 gives the absorption coefficients of different kinds of glass, from which the length of capillary necessary to prevent radiation from passing through the walls may be obtained.

Table 1. Linear Absorption Coefficients for Cu K$\alpha$ Radiation

| | cm$^{-1}$ |
|---|---|
| Soda glass (Chance G.W.I.) | 104 |
| Pyrex glass (Hysil) (Chance G.H.I.) | 70 |
| Lead glass (Chance 30% Pb) | 210 |

---

* The late J. N. Kellar was solely responsible for the early stages in the development of the micro-beam technique.

For lead glass capillaries, when the excitation voltage does not exceed 40 kv., a length of 2 to 3 mm. is sufficient to prevent appreciable transmission through the walls of the tube of the radiation produced. By drawing out the capillaries, bores down to 1 $\mu$ in diameter have been obtained. The tubes need not be perfectly straight, as the radiation may be transmitted by total reflection from the walls.

Small lead pinholes (down to approximately 6 $\mu$ in diameter) have been made by pressing lead around etched tungsten wires and subsequently withdrawing these. The apparatus for producing these pinholes is similar to one described by MacArthur (MacArthur 1945, see also Hirsch 1950) for pressing lead around steel needles.

### *The Camera*

The camera consists essentially of a double optical bench which carries a number of different components. The various components may be brought as close to each other as is desired by placing them alternately on opposite benches. Each part fits into a small slide which can be moved along the bench and locked to it at any point. The slides are provided with stops so that the components can be removed and replaced again in exactly the same position. All parts of the camera have been machined very accurately. The components include two collimator holders, of the type shown schematically in Figure 1, which provide tilting and traverse adjustments for the pinholes or glass capillaries. The plate A is locked into a slide on the optical bench, and the plate B is held against the three tilting screws C by springs D. E slides on B, and is pressed against it by the spring F. The screw G controls the horizontal traverse. H provides a vertical movement by rotating the slide E about the pivot I, E being held up by the spring J. The brass tube K is fixed to E and holds the capillary L.

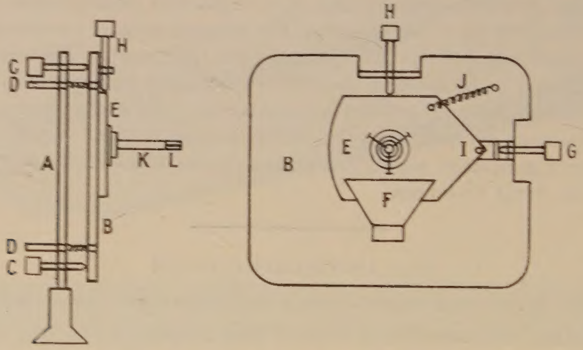

Figure 1. Schematic diagram of collimator holder.

Generally only one collimator is used, the divergence being controlled by the size of the focal spot on the target of the x-ray tube, but for accurately parallel beams both collimators may be necessary.

Two film holders can be used for simultaneous transmission and back reflection photographs. Warping during exposure may be avoided by covering part of the film with a brass plate or by using a photographic plate.

Other components of the camera include a specimen holder, a telescope for observation of preselected areas on the specimen, and a lid for evacuation or filling the camera with hydrogen. A general view of the camera is shown in Figure 2. Further details are given by Hirsch (1950).

### *Aligning of Capillaries*

The capillary is brought into approximate alignment with the aid of a Geiger counter and counting rate meter, the collimator holder being adjusted until a maximum reading is obtained on the meter. (It is advisable to keep the voltage on the x-ray tube low during alignment to reduce any short-wavelength radiation transmitted through the walls of the capillary.) Photographs taken at a distance from the capillary equal to that between the capillary and the x-ray tube focus should show an approximate image of the focus. Any small mis-setting can be recognized by a halo, due to the total reflections from the walls, associated with the spot on the film. The capillary is in correct alignment when the halo has disappeared. Figure 3 (see Plate*) shows a typical sequence of setting photographs.

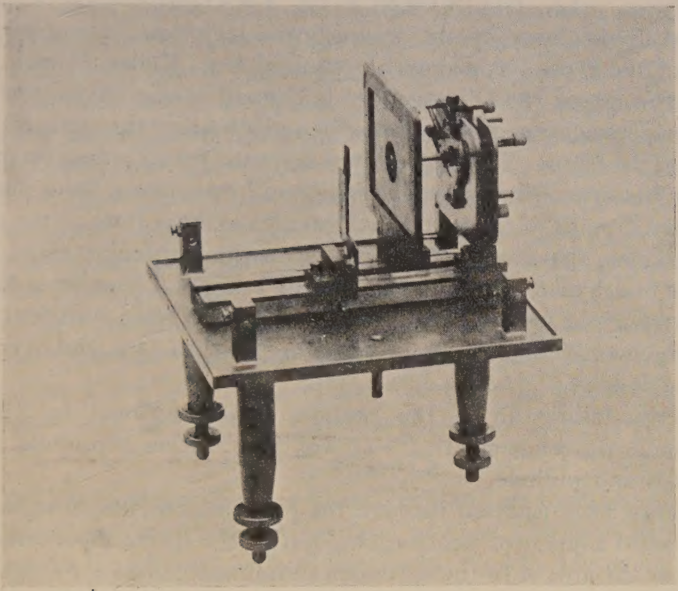

Figure 2. General view of camera, showing double optical bench, collimator holder, back reflection plate, and specimen holder.

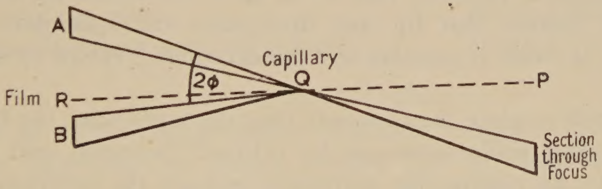

Figure 4. Section through centre of focus and capillary.

The geometry of the reflection halo can be understood from Figure 4, showing a section through the centre of the focus P, the capillary Q, and the film R. A is a direct image of the focal section in this plane, while B is due to a single reflection from the walls of the capillary. The angle of reflection $\phi$ in this plane is determined by measuring the angle AQB $=2\phi$. When $\phi$ is known, the necessary adjustment to the capillary can be estimated.

---

* For Plates see end of issue.

If in any plane $\phi$ exceeds the maximum angle of total reflection, the halo is incomplete (Figure 3). The maximum angle of total reflection can be determined very simply by measuring $\phi$ at the point at which the halo disappears. Its approximate value was found to be 15′ for lead glass capillaries (for Cu K$\alpha$ radiation).

## § 3. DIVERGENCE OF X-RAY BEAM

In many applications of the micro-beam technique a knowledge of the divergence of the x-ray beam may be required (e.g. Kellar, Hirsch and Thorp 1950). In this paper the 'divergence' is defined as the angle over which any point on the specimen receives radiation in a given plane through the point and a line section of the focus. This angle is not the same for all points on the specimen in this plane, because of the presence of the capillary between focus and specimen. Also, for a given point, if the apparent focal area and the bore of the capillary are not truly circular, the divergence depends on the particular plane considered. It is possible to calculate the divergence of the beam at any point in any direction if the geometry of the arrangement is known, but it is often sufficient to calculate a mean divergence, averaged over the whole area of the beam, and over all possible directions, as follows:

The energy falling on to the pinhole is proportional to $FP/r_0^2$ where $F=$ area of focus (foreshortened), $P=$ area of cross section of pinhole, $r_0=$ distance between focus and pinhole.

This energy is transmitted through the pinhole and falls on to the specimen. If $d\omega$ is the solid angle over which each point on the specimen receives radiation, averaged over the area $A$ of the specimen illuminated, $A d\omega = FP/r_0^2$.

If $d\omega$ is replaced by a right circular cone of semi-angle $\frac{1}{2}d\theta$, we have $A\pi(d\theta)^2/4 = FP/r_0^2$ or $d\theta = 2(FP/\pi A)^{1/2}/r_0$. If the pinhole is circular, of diameter $d$, $d\theta = d(F/A)^{1/2}/r_0$. Thus a mean divergence $d\theta$ can be calculated directly from the focal area and the area of the specimen illuminated (determined from pinhole photographs), and is independent of the shapes of these areas.

The divergence of the beam may be varied conveniently by altering either the size of the focal spot $F$ or the specimen–focus distance $r_0$. It is shown in II that the intensity of x-rays per unit area of focus increases as the area of the focus is decreased. Hence, for a given divergence of the beam at the specimen, the focus should be as small as possible, that is the specimen–focus distance should be a minimum. It follows that for any divergence the specimen–focus distance should be kept as small as possible and the divergence varied by altering the size of the focus.

If it is difficult to place the specimen near the x-ray tube, the totally reflecting properties of glass collimators can be utilized (Jentzsch and Nähring 1931, Nähring 1934). By placing the collimator between the specimen and the x-ray tube the divergence may be increased to twice the maximum angle of total reflection for the glass. The increase of intensity is entirely due to the increase of divergence. For a beam of given divergence some improvement of intensity can be obtained if, by using totally reflecting collimators, the size of the focus can be decreased.

However, totally reflecting collimators usually cause non-uniformity of divergence. Hence, in the present technique, the reflecting properties of the capillaries are not used; the capillaries serve merely to limit the area of specimen illuminated.

### *Divergence, Resolution and Exposure*

The divergence of the beam controls the angle over which reflection can take place, which affects both resolution and exposure of the photographs. The optimum divergence depends on the nature of the experiments. If single reflections from stationary crystals are observed (Kellar, Hirsch and Thorp 1950), the divergence must be as small as possible for maximum resolution. For minimum exposure the divergence of the beam must be just sufficient to cover the range of angles over which the crystal can reflect appreciably. A smaller divergence results in loss of radiation, since the crystal cannot reflect over its whole range; a larger divergence leads to loss due to the increased size of the focus and consequent decrease of intensity of X-rays per unit area.

For shortest exposure the film should be placed as near to the specimen as possible, but a lower limit is set by the size of the film grain. Little advantage is gained by using fine grain films; the total number of film grains exposed per spot should be the same whatever film is used.

On the average, crystals may be taken to reflect over approximately $10^{-3}$ radians, and since the specimen–X-ray tube distance cannot usually be reduced below about 2 cm., the lower limit of foreshortened focus is approximately $20\,\mu$ for optimum exposure in this type of experiment.

## § 4. EXAMPLE

Figure 5 (*a*) (see Plate) shows a back reflection photograph of cold-worked aluminium examined with the normal type of back reflection camera. If the same specimen is illuminated with a micro-beam ($\sim 35\,\mu$ diameter) a spotty ring photograph is obtained (Figure 5 (*b*)). Typical conditions for the photographs are:

| | |
|---|---|
| Diameter of capillary | $35 \times 10^{-4}$ cm. |
| Length of capillary | 0·3 cm. |
| Divergence $d\theta$ | $\sim 10^{-3}$ to $3 \times 10^{-3}$ radian |
| Specimen–film distance | 0·9 cm. |
| Distance between specimen and end of capillary | 0·2 cm. |
| Size of spots | $\sim 50 \times 10^{-4}$ cm. |
| Size of film grain | $\sim 5 \times 10^{-4}$ cm. |
| Size of reflecting crystallite | $\sim 2 \times 10^{-4}$ cm. |
| Exposure (Cu K$\alpha$ radiation) | 10 hours |
| Loading of X-ray tube | 20 ma., 40 kv. |
| Size of foreshortened X-ray tube focus | $0{\cdot}015 \times 0{\cdot}015$ cm$^2$. |

## § 5. CONCLUSIONS

It is clear from the above results that this technique can be used with polycrystalline aggregates to obtain photographs of single reflections from crystals of diameter $2\,\mu$ in a reasonable time. It follows that the technique may also be used to obtain photographs of a single reflection from an area of the same order of size on the surface of a large single crystal or from a small isolated crystal fragment.

## ACKNOWLEDGMENTS

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It is a pleasure to acknowledge the help we have received from Mr. C. E. Chapman in designing the micro-beam camera. The performance of the camera s a measure of the extreme precision with which Mr. Chapman carried out its construction.

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# An X-Ray Micro-Beam Technique: II—A High Intensity X-Ray Generator

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*ABSTRACT.* A high intensity x-ray generator has been constructed which combines the use of small foci and a rotating anode. Improvements in target loading per unit area of about fifty times compared with ordinary stationary target tubes* have been obtained. Any width of focus between about 1 mm. and 50 $\mu$ can be produced by means of an adjustable, biased electron gun. A detailed experimental investigation of the focusing properties of the high voltage gun has been made; the conditions for obtaining small foci and maximum current are derived. The magnitude of the negative bias is found to be the most important factor controlling the focusing action of the electron gun. The effect of bias on focal structure is discussed for several types of filament, and is explained on the basis of suppression of emission from different parts of the filaments.

## I. CONSTRUCTION AND PERFORMANCE OF GENERATOR

### §1. INTRODUCTION

The nature of the micro-beam experiments required the production of x-ray beams of high intensity and small variable divergence. In these experiments the divergence of the x-ray beams can be controlled most efficiently by altering the size of the focus on the x-ray tube target, while keeping the distance between the specimen and the focus as small as possible. (For further details see Hirsch and Kellar 1951, to be referred to as I.) The small divergences necessary require smaller foci than those commonly used. For these small foci the target loading per unit area (and hence the x-ray intensity per unit area) can be considerably increased relative to a tube with a focus of normal dimensions. An electron gun has been designed with which the focal dimensions can be altered over very wide limits and, in particular, can be made very small. A simultaneous increase in x-ray intensity is obtained for all focal sizes by the use of a large rotating anode.

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* Commercially available x-ray tubes with loading of 0·1 kw/mm$^2$.

The generator is described in Part I of this paper. Part II includes a detailed experimental investigation of the focusing properties of the electron gun, and a discussion of the mechanism of focusing.

## § 2. APPARATUS

### (i) *The Rotating Anode*

The rotating anode consists of a 10 in. diameter water-cooled disc, as shown in Figure 1. A Wilson (Gaco) seal at A maintains the vacuum round the rotating shaft. A second seal is situated at B and the space between A and B is filled with vacuum oil for lubrication purposes. The Wilson seals have been found very satisfactory; their lifetime is several hundred hours (cf. Taylor 1949); however, great care must be taken in handling them, lest any scratches are made on the

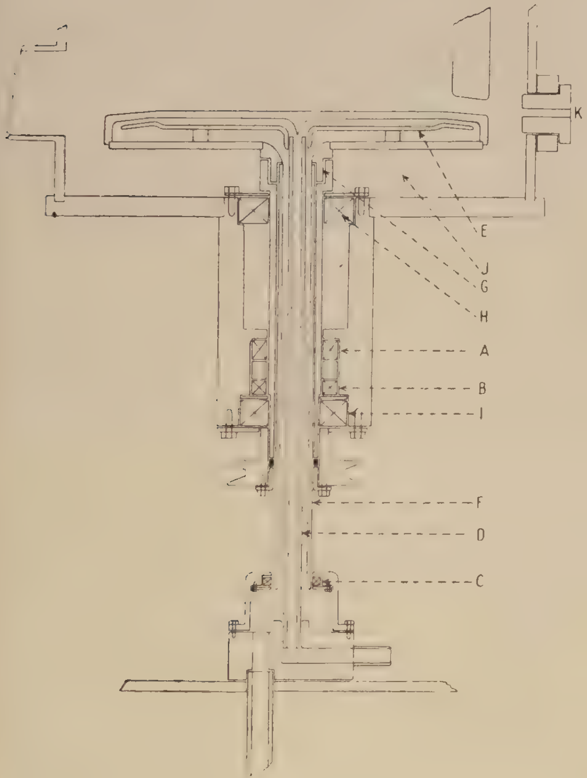

Figure 1. The rotating target.

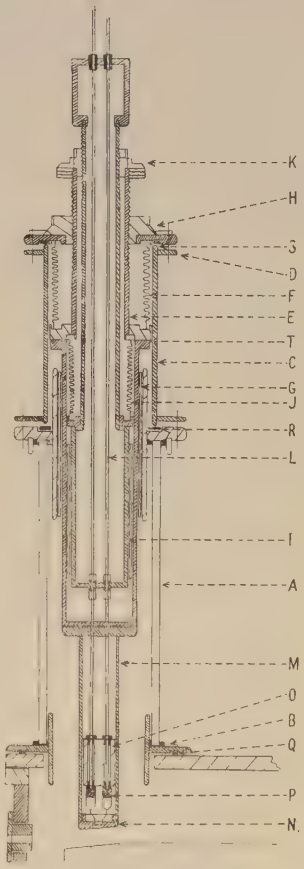

Figure 2. The electron gun.

knife-edge at which the seal is made. The rotating shaft requires careful machining and polishing. Another Wilson seal at C serves as the water seal.

The anode is rotated at about 1,450 revolutions per minute. The targets can be changed by unscrewing them, at G, from the rotating shaft which passes through the seals and bearings situated at H and I respectively. The anode is mounted in the vacuum chamber J with the plane of the disc horizontal. The rim of the anode is bevelled off at 6° and the x-ray beam emerges horizontally through the window K.

The rotating anode was designed and made by the Metropolitan-Vickers Electrical Co. Ltd. A similar type of anode, also employing Gaco seals, has since been designed by Taylor (1949).

### (ii) *The Electron Gun*

The electron gun was designed by the late J. N. Kellar to give very small foci and to allow changes in focal size to be made without breaking the vacuum. These features are obtained by the design of the grid (focusing hood), by insulating the filament so that a negative biasing voltage can be applied to the grid, and by the provision of two vacuum bellows mechanisms which permit precision movement of the filament relative to the grid and of the whole system relative to the anode.

The essential parts of the gun are shown in Figure 2. The gun is supported vertically above the bevelled rim of the target by means of a glass insulator A. A limited range of rotation about the vertical axis of the electron gun for alignment purposes is made by means of slotted flanges B. At the upper end of the glass insulator, permanently occupying a fixed position relative to the target, is a cylindrical housing C which is flanged at each end. At the upper end of this housing is another flange D to which the main body of the gun E is attached by the bellows F. The lower part of the main body fits in a keyway G; the upper part is threaded so that by rotating the graduated disc H the whole unit can be moved relative to the fixed housing C. A second cylinder I fits inside the main body and is attached to it by a second bellows J. This cylinder also moves in a keyway, and its position relative to the main body is controlled by the graduated disc K. The keyways prevent rotation of the filament or grid during adjustment. The filament leads L, of copper rod, are fixed inside the inner cylinder I but are insulated from it. A stainless steel tube M, at the end of which the grid cap N is inserted, is screwed to the lower end of the main body. Inside this tube steel filament rods are supported by the block O and terminate in the pin-chucks P in which the filament is held. The filament leads are insulated from the rest of the gun by means of small heat-resisting ceramic insulators, which are capable of withstanding at least six thousand volts. Rubber gaskets are used as demountable vacuum seals at the points Q, R, S and T. The apertures in the grid caps are generally in the form of simple slots. Line filaments, consisting of a single strand of tungsten wire, are used.

During the operation of the tube thermal expansion of the electron gun may cause the filament to move relative to the grid, and hence the focal size may alter during the exposure. This has been overcome by cooling the gun with a continuous current of air. After an initial warming-up period the size of the focal spot remains constant.

The focusing properties of the electron gun are described in Part II of this paper.

### (iii) *The Vacuum System*

The pumping system consists of a diffusion pump (British American Research Co., Type H.2), backed by a rotary pump (Metropolitan-Vickers, Type DR1). The filament life was found to be about the same whether oil or silicone was used as pumping fluid. With both, however, a layer was formed rapidly on the target at low power inputs, but at higher loadings flaked off, causing instability and limiting the life of the filament. The layer is thought to be due to the decomposition of the pumping-fluid vapour.

### (iv) *The Electrical System*

A Variac-controlled 5 kw. power pack, which can supply either A.C. or D.C. potentials, has been constructed. The D.C. potential is provided by a smoothed half-wave rectification circuit including a Philips gas rectifier (Type G.200). The biasing voltage is obtained by passing the tube current through a variable resistance between the filament and grid. This leads to self-stabilization of the tube current and the focal size. A series of protective relays is included.

## § 3. PERFORMANCE OF GENERATOR

### *Improvements in X-Ray Intensity expected*

The exposure time necessary in many micro-beam problems is determined by the X-ray intensity and hence the loading per unit area of focus. The data given here have been calculated on the basis of the Müller formulae (Müller 1931, see also Oosterkamp 1948), which hold, at least to a first approximation, over the range of focal sizes considered (10 mm. × 1 mm. to about 1 mm. × 0·1 mm.). In the rotating-anode generator the diameter of the ring traced out by the focus on the target is 20 cm., and the speed of rotation of the target is 1,450 r.p.m. Values for the hot and cool target surface temperatures of 300° C. and 20° C. have been assumed.

(*a*) *Effect of rotation of anode.* The improvement in intensity due to rotation alone can be expressed in terms of the ratio $K_R$ of the loading per unit area for a given focus on the rotating target to that for the same focus on a stationary target. Figure 3 (*a*) shows a curve of $K_R$ plotted as a function of the focal width $w$ for foci whose length is ten times their width. (Such a focus appears to be square when viewed at an angle of six degrees to the target surface.)

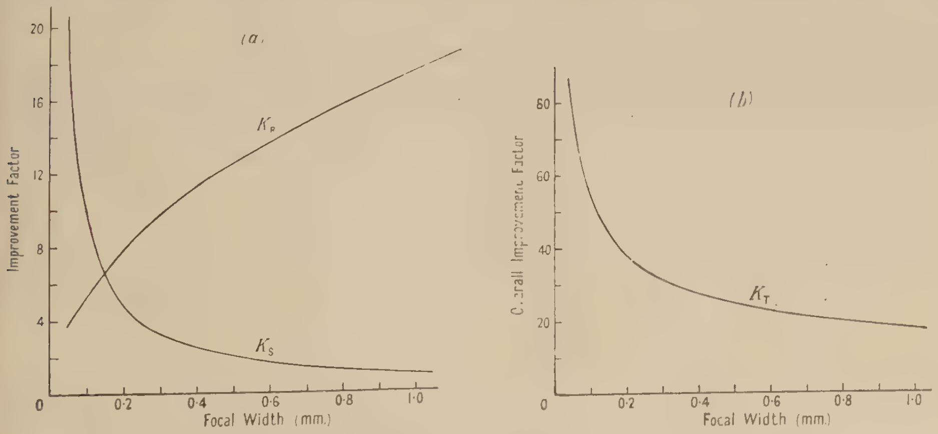

Figure 3. Theoretical improvement in target loading, (*a*) due to rotating anode ($K_R$) and due to small foci ($K_S$), (*b*) due to small foci on a rotating anode ($K_T$).

The improvement decreases as the focal width decreases, but is still quite substantial (about five times) for a focus of 0·1 mm.

(*b*) *Effect of reduction of focal dimensions.* A measure of the improvement due to the reduction in focal dimensions is given by the ratio $K_S$ of the loading per unit area for a focus of given dimensions on a stationary target to that for a focus of 10 mm. × 1 mm. on the same target. The latter loading corresponds to the case of an ordinary commercially available X-ray tube. Figure 3 (*a*) shows a curve of $K_S$ plotted as a function of the focal width $w$. The curve is again drawn for the case of a focus of length ten times its width. The value of $K_S$ increases rapidly as the focal dimensions are reduced.

(*c*) *Small foci on a rotating target.* It is convenient to use as a standard of reference a focus of 10 mm. × 1 mm. on a stationary target. The overall improvement in loading per unit area attainable for a small focus on the rotating target is then given by the factor $K_T = K_R K_S$. Figure 3 (*b*) shows a graph of $K_T$ plotted against the focal width $w$. It can be seen that for the foci most likely to be used, in the range of width about 0·1 mm., overall improvements in loading per unit area of about fifty times are to be expected.

### §4. DIMENSIONS OF FOCI AND LOADING VALUES OBTAINED

Any overall width of focus from about 1 mm. to $50\,\mu$ can readily be obtained (see Part II). At the usual operating voltages ($\sim$40 kv.) the electron gun is capable of supplying more beam current than is necessary to melt the target for every focal spot size within the limits given above.

A typical loading figure obtained for a fairly small rectangular focus is 25 ma. at 40 kv. on a focus of 1·5 mm. × 0·15 mm. (giving an apparent focus of 0·15 mm. square when foreshortened). This focus was obtained with an aperture of width 3·0 mm., thickness 4·0 mm. and length 8·0 mm. The filament, a 0·4 mm. diameter wire of length 4·0 mm., was placed about 2·0 mm. behind the front of the grid. The anode–grid spacing was about 6·0 mm. and the bias voltage was 1,450 volts. In this case the loading per square millimetre is 4·4 kw/mm$^2$. This represents an improvement in X-ray intensity per unit area of about fifty times compared with an ordinary stationary target tube, which agrees with the expected values, cf. Figure 3 (*b*). An estimate of the order of magnitude of the improvement in exposure time is given by the fact that high resolution back-reflection micro-beam photographs can be taken in about 10 hours with the generator. Low resolution photographs of the same specimens taken on an ordinary sealed-off tube needed exposures of about 200 hours.

Several high intensity X-ray tubes have been described in the literature. A review of the performance of some rotating-anode tubes may be found in a paper by MacArthur (1945). Some X-ray tubes with fairly small foci have also been described (Poittevin 1947, Goldzstaub 1947, Guinier and Devaux 1943). A simple fine focus tube employing a small circular focus on a stationary target has recently been described by Ehrenberg and Spear (1951). The present generator combines the use of fine foci and a rotating anode.

## II. THE FOCUSING PROPERTIES OF THE ELECTRON GUN

### §1. INTRODUCTION

A systematic investigation of the focusing properties of the electron gun has been carried out with the object of determining (*a*) how small foci can be produced, and (*b*) under what conditions maximum current can be attained in a given focus.

Line and strip filament systems were studied. For a given grid and filament experiments were made on the effect of a number of variables on both the focal size and the beam current attainable. These variables were: (i) biasing voltage of grid relative to filament, (ii) filament setting in grid: the distance between the front of the grid and the front of the filament is denoted by $s$, (iii) distance $d$ between the anode and the grid, (iv) high potential $V$ between the anode and the grid. The effects of the geometry of the grids and filaments have also been examined.

It was found that nearly all foci contained a banded structure, and this was studied to obtain some information about the mechanism of focusing.

## §2. VARIATION OF FOCAL SIZE

### (i) *Experimental Procedure and the Effect of Bias*

The experiments were carried out by taking a series of pinhole photographs of the focus for several values of bias, while all the other variables were kept constant. A 30 $\mu$ diameter glass capillary was used as a pinhole to obtain high resolution. The effect of a particular variable, other than bias, was investigated by taking such 'bias runs' for different values of this variable. The experiments described in this and the following sub-sections were performed with a plane grid with an aperture of width 3·0 mm., length 8·0 mm., and depth 6·5 mm. A straight line filament of length 5·0 mm. and diameter 0·4 mm. was used.

The effect of bias can be seen from any one of the curves shown in Figures 4, 5 and 6. Generally the focal width decreases with increasing bias voltage until a plateau is reached, where the rate of decrease is very small. At large bias voltages the foci again decrease in size. Similar curves have been recorded before (e.g. Guinier and Devaux 1943, Arndt 1948), but the decrease in focal width at very large bias values has not previously been reported. For large biasing voltages a limiting spot width is obtained, which is of the order of 50 $\mu$ for typical grids.

The curves were obtained at a constant current of 10 ma. until a saturation point was reached. At this point, although the filament-heating current was raised, it was no longer possible to obtain 10 ma. emission. Subsequent measurements were made at the appropriate saturation current. The occurrence of saturation limits the greatest value of bias voltage obtainable. The saturation points are indicated by crosses on Figures 4, 5 and 6.

### (ii) *Variation of Focal Size with High Tension*

Usually the focal width obtained at a given bias value increases with increasing high tension, but the limiting spot width is independent of $V$ over the range examined in the experiments (Figure 4).

### (iii) *Variation of Focal Size with Filament Setting*

The shapes of the curves alter as $s$ is increased and the plateaux on the curves become more pronounced. For a given bias voltage the focal width, and in particular the limiting focal width, decreases as $s$ increases (Figure 5).

### (iv) *Variation of Focal Size with Anode–Grid Spacing*

Figure 6 shows a group of curves in which the focal widths are plotted against bias voltage for different values of the anode–grid spacing $d$. As $d$ is reduced the curves are considerably altered, and the plateau on each curve becomes

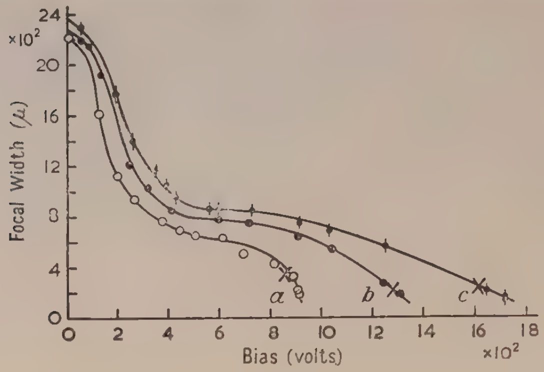

Figure 4. Variation of focal width with bias for various values of the high tension. (*a*) 25 kv., (*b*) 35 kv., (*c*) 45 kv. ($s=2{\cdot}0$ mm., $d=8{\cdot}0$ mm.). Crosses indicate saturation points.

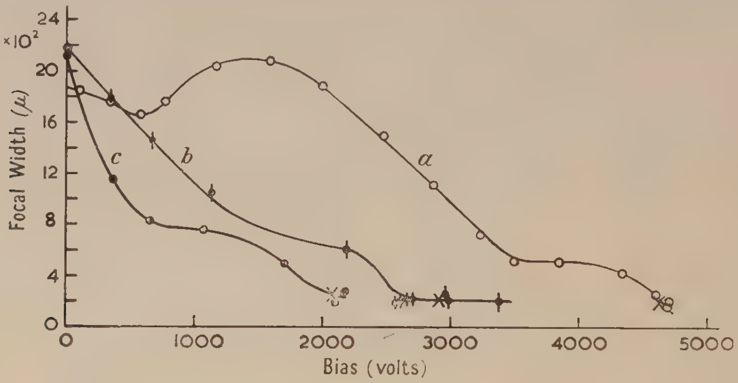

Figure 5. Variation of focal width with bias for various positions of the filament inside the grid aperture. (*a*) $s=1{\cdot}0$ mm., (*b*) $s=1{\cdot}4$ mm., (*c*) $s=1{\cdot}7$ mm. ($d=8{\cdot}0$ mm., 35 kv.). Crosses indicate saturation points.

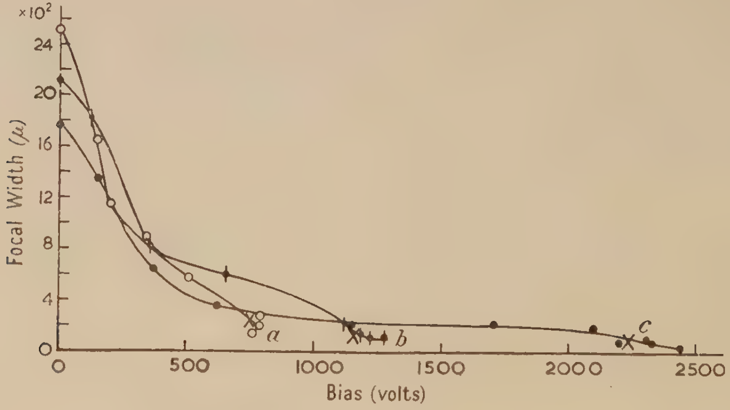

Figure 6. Variation of focal width with bias for various values of the anode–grid distance. (*a*) $d=11$ mm., (*b*) $d=8$ mm., (*c*) $d=6$ mm. ($s=2{\cdot}0$ mm., 35 kv.). Crosses indicate saturation points.

more prominent. For all values of bias voltage the focal widths are greatly influenced by the anode–grid spacing (cf. Guinier and Devaux 1943). The zero bias and limiting focal widths were particularly examined over a large range of values of *d*, and the results are shown in Figure 7. These curves show that both the zero bias and the limiting focal widths decrease with decreasing anode–grid spacing. A minimum focal width occurring at small values of $d$, as reported by Guinier and Devaux (1943), has not so far been observed, but no measurements have been taken with $d$ less than 4·0 mm. [Points with $d$ less than 6 mm. were obtained with a 'sloping' grid (see §5) and are not shown in Figure 7.]

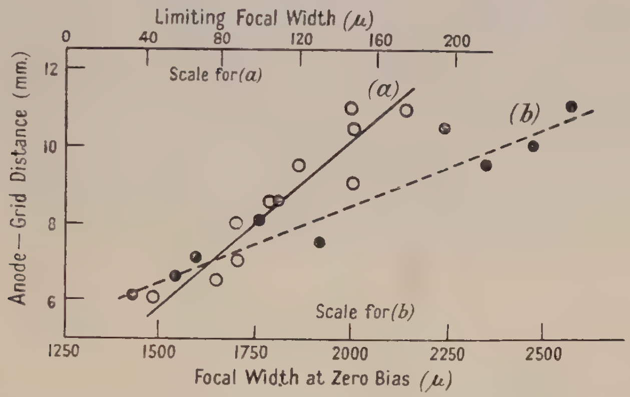

Figure 7. Variation of limiting focal width and focal width at zero bias with anode–grid distance. (a) Limiting focal width (25 kv., $s$=2·0 mm.), (b) focal width at zero bias (25 kv., $s$=2·0 mm.).

## § 3. VARIATION OF AVAILABLE BEAM CURRENT

Figure 8 shows a set of curves in which the saturation beam currents are plotted as a function of the filament setting for various values of the high tension. These were derived from curves similar to those of Figure 4 for given values of the focal width and the anode–grid spacing. It follows from Figure 8 that the smaller the value of $s$, the more current can be obtained in a given focus. Further, the available beam current increases with increasing high tension, and with decreasing anode–grid spacing.

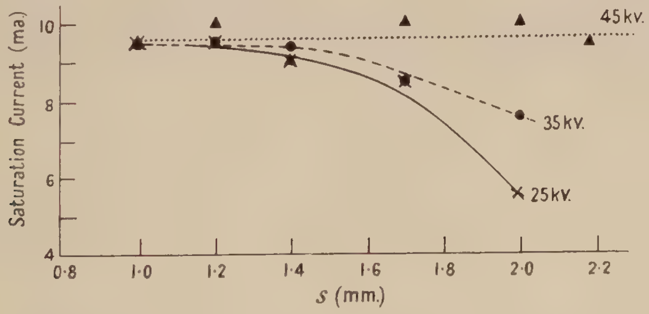

Figure 8. Variation of saturation current with $s$ for a given size of focus and value of $d$. Width of focus 0·2 mm., length of focus 3·0 mm., $d$=8·0 mm.

## § 4. OPTIMUM FOCUSING CONDITIONS

The results given in the preceding sections lead to the following optimum focusing conditions. The maximum beam current for any given focal size is attained if the anode–grid spacing is a minimum and the high tension is a maximum. The filament should be as far forward as possible in the grid aperture, and the bias voltage should be made correspondingly large. The value of $s$, however, cannot be decreased indefinitely, as the limiting focal size tends to increase. Further, at very small values of $s$ the bias voltages necessary to give the required focal dimensions may become prohibitively large for the insulation between the filament and the grid.

## § 5. FOCUSING GRID GEOMETRY

The results given above were obtained for a plane grid with a parallel-sided aperture, as shown diagrammatically in Figure 9 (*a*). With grids of this type the length of the filament can easily be set accurately parallel to the plane of the grid surface. This condition leads to the most uniform intensity distribution along the length of the focus. The parallelism of filament and grid surface is more difficult to obtain with grids which are parallel to the bevelled rim of the target (Figure 9 (*b*)). It was found, however, that such sloping grids gave results similar to those obtained with plane grids. Except when small values of the anode–grid spacing are required, plane grids can therefore be used.

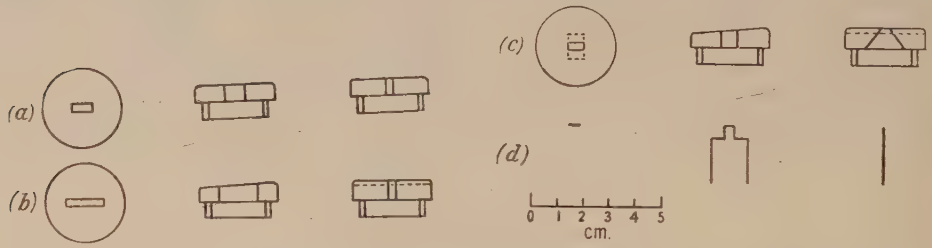

Figure 9. Grid-cap geometry.
(*a*) Plane grid with parallel-sided aperture, (*b*) sloping grid with parallel-sided aperture, (*c*) sloping grid with wedge-shaped aperture, (*d*) line filament.

Experiments carried out on grids with apertures of different widths showed that, for a given width of focus, more current can be obtained by increasing the aperture width and the bias voltage. The limiting focal width, however, increases with aperture width. The effects of changes in aperture width are similar to those of changes in the filament setting.

Wedge-shaped apertures (Figure 9 (*c*)) have also been studied. Their properties correspond to those of parallel-sided apertures of rather greater width than the front of the wedge.

## § 6. FILAMENT DIMENSIONS

An investigation of line filaments of different diameters showed that more current can be obtained for a given focal width with filaments of larger diameter. Strip filaments yield very uniform foci under light bias conditions, but the current available is limited. The length of the focus obtained is determined largely by the length of the filament, but is also influenced by the length of the aperture and the other gun variables.

## § 7. STRUCTURE OF FOCI

With the line filament systems investigated the foci have almost invariably an undesirable banded structure (Thorp 1950). Some typical sequences of focal patterns obtained for increasing bias voltage are shown in Figure 10 (Plate*). There are usually two systems of bands, lying respectively parallel and perpendicular to the length of the filament, but the former are generally more prominent.

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* For Plates see end of issue.

The bands parallel to the length of the filament become less distinct under high bias conditions. Thus fairly uniform small foci may be produced at large bias values. At intermediate bias values the foci have always very intense edges and often show sharp central bands. The focal structure is clearly resolved in these experiments, though it has not been mentioned by previous workers.

The bands perpendicular to the length of the filàment are usually much less pronounced, and under light bias conditions are confined mostly to the edges of the foci. These bands may be due to emission from the legs of the filament, or due to a focusing effect at the edges of the aperture. At large bias values these bands may constitute the only structure in the focus.

For the larger ranges of focal size, foci showing the smallest amount of structure were obtained by using strip filaments under light bias conditions.

Line filaments were used to obtain greater uniformity of electron density along the length of the focus than is possible with the conventional type of helical filament.

## § 8. MECHANISM OF FOCUSING

### (i) *The Effect of Bias*

A three-hairpin filament was placed in a long aperture so that the hairpins were at slightly different distances from the front of the grid (Figure 11). Emission from such a filament will be confined in the main to the hairpins alone. At zero bias the focus is found to contain three well-defined bands due to the three hairpins. When bias is applied, the intensities $I$ of the bands decrease in the order $I_c < I_b < I_a$. These focal changes are illustrated in Figure 12 (Plate). It seems from this experiment that the application of bias reduces the number of electrons drawn off from the filament.

In the absence of bias the field at the filament due to the potential between the anode and the filament will decrease progressively as $s$ is increased (this field is afterwards called the anode field). Further, in the absence of the anode potential, the fields due to bias will be the same at the three hairpins (this field is afterwards called the bias field). It follows from the experiments that the effect of a given bias field in limiting the emission from this filament is greatest at points of weakest anode field.

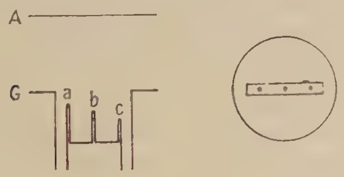

Figure 11. The hairpin filament. A=anode, G=grid: sections through and perpendicular to the plane of the filament.

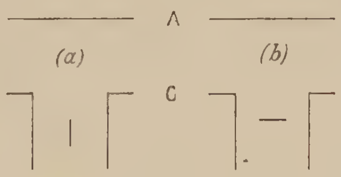

Figure 13. Geometry of strip filament systems: sections perpendicular to the length of the filament. A=anode, G=grid.

This result can be expected from *a priori* reasoning. The bias field partially neutralizes the anode field at the filament. The effective emission from any point of a filament decreases as the resultant field is reduced, and is completely suppressed when the field becomes zero (except for electrons with high thermal energies). For any filament system the application of bias may be expected to suppress first the emission at those points of constant bias field at which the anode field is weakest. The greater the bias the larger becomes the area of the filament at which the resultant field is zero or negative, and hence the total current drawn off decreases. (A discussion of the effect of bias on the emission from a plane cathode in an immersion lens is given by Jacob (1950).) In the light of these considerations the effect of bias on some of the filament systems used can be discussed.

The focusing action of an electron gun of similar geometry has been discussed by one of the authors (Thorp 1949, 1950) for the particular case of the filament and grid at the same potential.

### (ii) *Interpretation of Focal Patterns*

(*a*) *Strip filaments.* In interpreting the effects of bias on the focal patterns it is necessary to know which parts of the focus are due to electrons emitted from particular regions of the filament. With strip filament systems the simple form of the anode field enables reasonable assumptions to be made about the nature of this correlation. Experiments were carried out with the two strip systems shown in Figure 13. In the discussions which follow a plane section perpendicular to the length of the filament is considered.

*Planes parallel to sides of grid aperture* (Figure 13 (*a*)). The focus at zero bias contains a sharp central band and very broad sidebands ($s=1\cdot5$ mm.). On the application of bias the extent of the outer bands is reduced until only the sharp central band remains (Figure 14 (*a*), Plate). The central band probably arises from the front edge of the strip, and the sidebands from the sides of the strip. This system is particularly simple in that the anode field can be expected to decrease with increasing $s$ along the plane of the strip, while the bias field is approximately constant over a considerable part of it. Thus on application of bias, emission from the areas at large values of $s$ will be suppressed first; this explains the suppression of the sidebands.

*Plane perpendicular to sides of grid aperture* (Figure 13 (*b*)). The focus at zero bias contains a broad central band flanked by two smaller bands ($s=1\cdot5$ mm.). On the application of bias the extent of the sidebands is reduced until only the broad central band remains. On further increase of bias the width of the central band is reduced (Figure 14 (*b*), Plate).

At the front of the strip the anode field will be fairly uniform and normal to the plane of the strip over a considerable area, so a broad central band would be expected. At the back of the strip the anode field will be small, and the narrow sidebands are probably due to emission from this part. The previous arguments and this assumption lead to the conclusion that, on the application of bias, emission from the back of the strip will be suppressed first. This explains the initial suppression of the sidebands. Further increase of bias will reduce the area on the front of the strip from which most of the electrons are drawn, and this accounts for the decrease in the width of the central band.

(*b*) *Line filaments.* Here the anode field varies in a complicated manner round the filament. The approximate areas of the filament contributing to different parts of the focus have been studied previously for the zero bias case (Thorp 1950). These areas are a function of the position of the filament within the aperture.

With the filament at $s=1\cdot5$ mm. a three-banded focus is expected. The outer bands are due to electrons from the sides of the filament, while the central band has contributions from the front and back. The anode field is least at the back of the filament, and increases progressively towards the front. The bias field is symmetrical with respect to a plane passing through the centre of the filament parallel to the front of the grid. The bias field is least at the front and back of the filament and greatest at the sides. Thus for a given value of bias emission will be suppressed from a greater area of the rear half of the filament than of the front; it cannot easily be determined, however, whether the suppression will begin at the sides or the back of the filament. The experimental focal changes are shown in Figure 10. The immediate effect of the application of bias is a decrease of focal width. On the above basis this corresponds to a reduction of the emission from the sides of the filament. For larger bias values the central band is reduced in intensity relative to the rest of the focus, and this is thought to correspond to the suppression of the electrons from the back of the filament. Finally, only a small focus is left, due to electrons from the front of the filament.

With the filament further forward it appears from the theoretical work that, at zero bias, electrons from the back of the filament contribute to the intensity at points between the centre and sides of the focus. The experimental curve for $s=1{\cdot}0$ mm. (see Figure 5) shows that for a large range of bias values the focal size is approximately constant; however, large changes of structure within the focus take place. This is thought to be due to suppression of electrons from the back of the filament. Only for very large bias values is the overall focal size reduced; this corresponds to a limitation of the number of electrons drawn off from the sides and front of the filament.

## § 9. CONCLUSIONS

For all the filament–grid systems considered above the bias field is zero along a line through the centre of the aperture and parallel to its sides. It may be expected, therefore, that, whatever the value of the bias potential, there will always be a point on the filament at which the resultant field is finite and from which electrons are drawn. The limiting foci are then due to emission from very small areas of the filament.

In summarizing the results it may be stated that the focal patterns discussed above can all be explained on the basis of the two following principles: (i) the focal structure at zero bias is due to a grouping of electrons emitted from different parts of the filament, (ii) on the application of bias the focal structure changes are due to the suppression of electrons drawn off from different parts of the filament.

## ACKNOWLEDGMENTS

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# Recovery and Recrystallization in Highly Stressed Pure Aluminium

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*ABSTRACT*. The paper contains an account of an x-ray study of the phenomena observed when polycrystalline pure aluminium is subjected to heavy compressional stresses. The rate of recovery to the metastable state under various conditions has been examined in some detail.

During recovery recrystallization occurs; the crystals grow rapidly at first, but afterwards the rate of growth decreases until finally, after the metastable state has been reached, there are no further changes in crystal sizes. The recovery phenomenon and recrystallization are two different processes but they are not independent of each other. Crystals grow when the value of the stress varies from point to point in the material; when the stresses become evenly distributed, as is found to be the case in the metastable state, the crystals stop growing.

The change from the state immediately following plastic deformation to the equilibrium crystalline state takes place in two stages, one brought about by strain energy and the other by thermal energy. The results are considered in the light of the dislocation theory.

## § 1. INTRODUCTION

Compressed aluminium of 99·998% purity after release of stress has in general a lattice parameter value different from that of material in the equilibrium state. In course of time the parameter changes according to an exponential law, finally reaching a constant value which is higher than the equilibrium value. It is then said to be in the metastable state. If the material after reaching this state is annealed at high temperature, say 500° c., for a suitable period of time, the lattice parameter decreases from the metastable to the true equilibrium value. These results were described in detail in a previous paper (Owen, Liu and Morris 1948). The experimental arrangement was such that the x-ray beam fell normally on the surface of the specimen, that is, in the direction in which the stress was applied.

In the previous investigation the lattice spacings were calculated mainly from the measurements of lines in structure spectra obtained with a Debye–Scherrer camera; the present work was carried out almost entirely with the aid of a back-reflection x-ray camera so that lattice spacings could be determined without having to resort to extrapolation. This course was adopted because it was desirable to obtain information concerning the behaviour of aluminium over the period between the release of stress and the attainment of metastable equilibrium when it might not behave isotropically. The new arrangement enabled interplanar spacings to be measured directly, and these measurements could be made with stationary specimens so that the direction of stress relative to the reflecting planes could be specified.

The immediate aim of the work was to investigate whether in heavily compressed aluminium consisting initially of a polycrystalline aggregate, any difference could be detected in the rate of recovery * after deformation when the planes reflecting the x-rays made different angles with the direction of the applied stress. Greenough (1949) had found with material subjected to tensile stress that different positive and negative strains existed in different sets of planes.

It was also desired to study recrystallization in relation to the recovery process, and to examine the phenomena observed in the light of the dislocation theory.

To study the recovery process with polycrystalline specimens it seemed desirable to start with severe deformation, sufficient to produce a high degree of preferred orientation. The results therefore apply only to severely compressed specimens. The work differs from, and is complementary to, that of W. A. Wood and his collaborators (Wood 1939, 1941, 1948, Smith and Wood 1940, 1941 a, b, 1942 a, b, 1944) who have devoted attention mainly to materials subjected to tensile stress.

## § 2. EXPERIMENTAL PROCEDURE

The back-reflection camera employed for the work was constructed to a new design.† With it values of lattice spacings could be obtained to a high degree of accuracy from measurements made on reflections from one set of crystal planes. The x-ray photographs could be taken either when the plate specimen was kept stationary or when it was simultaneously oscillated and rotated. The continuous diffraction lines obtained by the latter method could be measured to a high degree of accuracy—a slightly higher accuracy than that attained in measurements of the diffraction lines in photographs obtained with stationary compressed specimens.

With the aid of a template the face of a specimen could be mounted over the centre of the camera to an accuracy of about ± 0·01 mm. The distance between the specimen and the film could also be determined to the same accuracy with this template. In much of the work which will be described the specimen and x-ray tube were kept undisturbed throughout a series of exposures so that the relative accuracy from exposure to exposure in any one series was very high, as the only errors are those arising from arc measurements. With cobalt radiation the (420) reflections from aluminium give sharp lines in a favourable position on the film; chromium and iron radiations were also used as they give well resolved lines which yield accurate values of the lattice spacings.

In addition to the measurement of the diameters of diffraction rings, measurements were also made on several spots observed in photographs taken

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* The term ‘ recovery ’ is used in several connections. Here it refers to changes in residual stresses present after the release of the applied stress, as observed by x-rays.

† A detailed account of the instrument will be given elsewhere.

at different times after the release of the stress. Each spot could be kept under observation and a record made of its change of position and shape in the course of time, this being possible since the incident x-ray beam was kept in a constant direction relative to the specimen.

In the back-reflection photographs reproduced in this paper it will be observed that two circles with very fine lines are imprinted on each film in addition to the diffraction rings. These serve as fiducial marks, the true diameters in different directions being accurately known; to facilitate the location of the spots on the diffraction rings, the outer circle is marked in degrees. A correction for the change in dimensions of the film in any radial direction, on processing, can be made by this device.

The specimens were in the form of plates between 1·5 and 2·0 mm. thick, some were square, others circular in cross section, the side or diameter being about 7 mm. They were given an initial annealing treatment at 500° c. for 21 days before compression and were irradiated near the centre of one of the flat faces.

## § 3. RESULTS

### (i) *Specimens Compressed in One Stage between Steel Plates*

(*a*) *Radiation of different wavelengths.* For this part of the work the x-ray beam fell normally on the surface of the specimen and in the direction of the applied stress. Three different radiations were used, namely, chromium, iron, and cobalt, these being reflected from the (222), (400) and (420) planes of aluminium. The specimen, after compression between two highly polished steel plates, was mounted on the camera and remained undisturbed while photographs with different radiations were taken. Typical photographs taken with cobalt radiation after a reduction of 62% in thickness are shown in Figures 1 to 8 (Plate I *). Figure 1 was taken half-an-hour after the stress had been released. In this photograph the (420) and (331) doublets appear as continuous diffraction rings showing the existence of a certain degree of preferred orientation amongst the crystallites. A few spots are also scattered sparsely over the rings showing that recrystallization had commenced at this early stage in material compressed by this amount. Passing along the series of photographs it is observed that the rings remain continuous but that the spots become more numerous. When the rings are studied closely it is found that after a certain time the pattern as well as the relative intensities of the spots become stationary. In the present instance this stage appears to have been reached in about 172 hours, but the pattern is almost the same in the photograph taken after 75 hours. The metal therefore reaches a stationary state; this is in accordance with previous findings (Owen, Liu and Morris 1948).

When the mean diameters of the diffraction rings were measured the curve between average percentage strain and time elapsed after release of stress shown in Figure 9 was obtained. The interesting fact that emerges from these measurements is that when the mean positions of the normals to the reflecting planes (222), (400) and (420) of aluminium from differently oriented crystallites are approximately in the directions of the stress, the value of the percentage strain in each case is the same and falls at the same rate for each plane until it reaches the final constant value of about $13{\cdot}7 \times 10^{-3}$ representing the residual strain in the material after reaching the metastable state. The curve in Figure 9 represents the recovery of material reduced 52·8% in thickness. In calculating the residual strain the equilibrium value of the aluminium lattice parameter was taken to be $4{\cdot}0406_5$ kx.

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* For Plates see end of issue.

(*b*) *Different angles between the x-ray beam and the direction of the applied stress.* Cobalt radiation only was used for this part of the investigation. The angle of incidence of the x-ray beam on the surface of the specimen was varied between 0° and 45°, but the experiment was carried out in sections with three different specimens; one was reduced 55·9% in thickness, the others 56·5% and 50%. In Figures 10 to 15 (Plate II) are reproduced some of the photographs taken with the specimen reduced 56·5% for the angles of incidence, 0°, 20° and 30°.

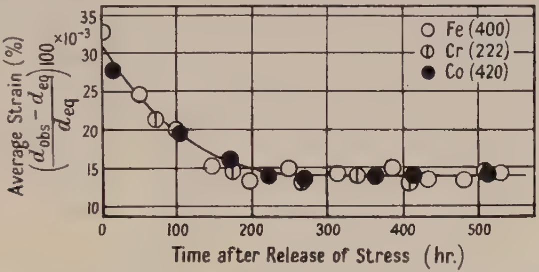

Figure 9. Relation between percentage strain and time elapsed after release of stress in aluminium reduced 52·8% in thickness in one stage. Normal incidence.

They show the state of the specimen a short time and a long time after the release of stress. The characteristics of these photographs are much the same as those in Figures 1 to 8. The crystallites show different degrees of preferred orientation in different directions according to the angle of incidence of the radiation on the surface of the specimen. Recrystallization is also not so vigorous as it was found to be after the 62% compression.

The results of the measurements of the photographs are shown graphically in Figure 16; the photographs taken after 55·9% compression are combined

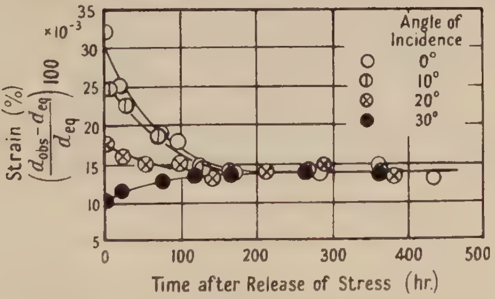

Figure 16. Relation between percentage strain and time elapsed after release of stress in aluminium reduced 56% in thickness in one stage, the radiation (cobalt) falling on the surface at different angles of incidence.

with those taken after 56·5% compression since the differences between the interplanar spacings at the same angle of incidence after these compressions are very small. The recovery curve is different for each angle of incidence chosen. A different set of crystallites is mainly responsible for each curve. The diagram shows that the (420) interplanar spacing undergoes different percentage changes according to the orientation of the planes to the direction of stress. The smaller the angle between the normal to the reflecting plane and the direction of stress, the greater is the initial percentage strain produced in a certain compression. The value of the measured percentage strain for all planes making a given mean angle with the surface of the specimen is characteristic of the mean angle of incidence of the radiation on the planes, and changes—it may increase or decrease—exponentially to the value of the final residual strain. The percentage strains for all angles of incidence reach the value of the residual percentage strain corresponding to the metastable state in approximately the same time after the release of stress. Hence the rate of recovery to the metastable state is faster the greater the initial percentage strain. There is a relation between the mean angle which the reflecting plane makes with the surface of the specimen, and the initial percentage strain, the latter tending to zero when the angle is about 45°. But the percentage strain increases for these planes in course of time owing to the influence of nearby strained crystallites, and eventually reaches the constant value corresponding to the residual percentage strain. The curves in Figure 16 also show that when the material enters the metastable state the values of the strains do not depend on the direction in which they are measured, as is the case in the early stages of recovery; the residual strain is now uniformly distributed and remains until it is removed by annealing the material at high temperature.

A series of exposures was taken with a specimen compressed to produce 47% reduction in thickness, when both the angle of incidence and the radiation were altered. The results are shown in Figure 17, and they corroborate those already discussed in the previous paragraphs.

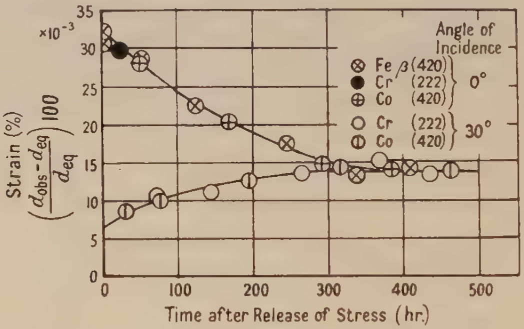

Figure 17. Relation between percentage strain and time elapsed after release of stress in aluminium reduced 47% in thickness in one stage, the radiation of different wavelengths falling on surface of specimen at angles of incidence of 0° and 30°.

### (ii) *Specimen Compressed in Two Stages*

Specimens which had been annealed at 500° c. for 21 days were cold worked in two ways: the first specimen was compressed in one stage to produce 62% reduction in its initial thickness; the second was compressed in the first stage to produce 60·6% reduction, and then after it had reached the metastable state, which it did in about 150 hours, it was given a second compression which produced a further 23·8% reduction in thickness so that its final thickness was 30% of its initial value.

In the first experiment the recovery curve, obtained with radiation falling at normal incidence on the specimen, followed the course to be expected from previous work; the percentage strain immediately after release of stress was $27{\cdot}5 \times 10^{-3}$, and decreased exponentially in about 150 hours to the value $13{\cdot}7 \times 10^{-3}$ corresponding to the metastable state. The first stage of the second experiment under the same conditions followed very closely the same recovery curve as was found in the first experiment, the initial strain being in this case $31{\cdot}2 \times 10^{-3}$. The initial percentage strain immediately after the release of stress in the second stage was found to be $22{\cdot}5 \times 10^{-3}$, and the material returned to the metastable state much more quickly than would be expected with fully annealed material compressed by the same amount. The results of the second experiment are shown graphically in Figure 18. The experiment illustrates the effect of work hardening produced by the first compression; to produce the same amount of reduction in thickness by compression in material already compressed, much more work is required than when the specimen is in the annealed condition. It is misleading to compare the amounts of deformation without considering also the initial condition of the specimen. The characteristics of the recovery phenomenon, however, which according to our previous view is governed by the strain set up in the crystallites under the influence of the 'plastic'* or transition layers, will be independent of the initial condition; the condition of the specimen before the application of cold work can only affect the amount of change in interplanar spacing and the rate of recovery. The foregoing results confirm this.

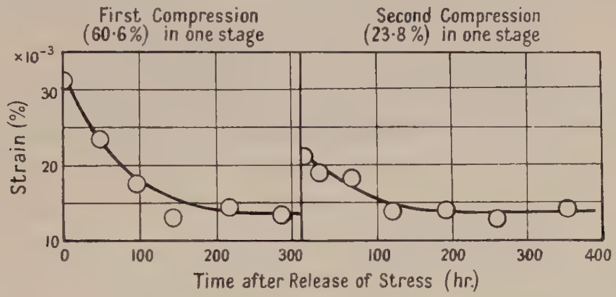

Figure 18. Recovery curves showing the effect of a second compression.

### (iii) *Etched Specimen*

With the object of investigating the condition of the material below the surface, a specimen reduced 55% in thickness by compression had its surface etched after taking the first photographs immediately after the release of stress. The surface was removed by electrolytic etching to a depth of about 0·1 mm. and the behaviour of the material examined as before. The recovery curve was found to be exactly the same as that obtained with an unetched specimen compressed by the same amount.

### (iv) *Specimens Compressed between Aluminium Plates*

In an attempt to find the condition of the material at greater depths, three specimens of the same size (10 mm. × 8 mm. × 2·4 mm.) which had been annealed at 500° c. for 21 days, were piled together and compressed between steel plates in the usual manner, the operation being performed in one stage. The surfaces of the middle plate B (Figure 19) and those of Plates A and C in contact with it were found to be uneven due to the reorientation of the crystal grains that occurs during compression.

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* The term 'plastic' in this connection is used in contradistinction to the term 'crystalline': in the latter state the material is elastic and obeys Hooke's law but in the former it is inelastic and does not obey the law.

with accuracy over the centre of the camera, so that the absolute accuracy of interplanar spacing determination would not be high; the relative accuracy, however, was unimpaired because the specimen once mounted was left undisturbed during a series of photographs. To increase the accuracy further,

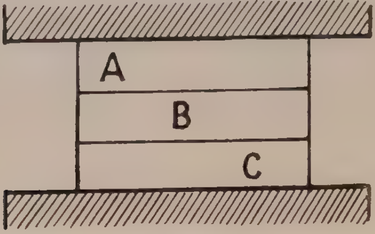

Figure 19.

Measurements were made on photographs taken with the middle specimen B. Owing to the unevenness of the surface it was difficult to mount the specimen a long slit was used with which an area of only 1·3 mm² of the specimen surface was irradiated. The recovery curve now obtained is shown in Figure 20 (lower curve) the radiation (cobalt) falling on the surface at normal incidence and reflected by the (420) planes. The form of the curve is different from that of the recovery curves obtained by the examination of the surface layers in contact with the steel plates; it is in fact very similar to the curve obtained with a filed surface (Owen, Liu and Morris 1948). But the final constant value of the spacing is so low compared with previous values that it was suspected that the specimen was incorrectly mounted, giving an erroneous value of the film–specimen distance. To check this a few photographs were taken with a circular spectrum camera, so that errors due to the eccentricity of the specimen on its mounting could be allowed for by extrapolation. This was legitimate procedure with material in the metastable state because the residual strain would then be evenly distributed. When approximately the same spot on the surface of the specimen was irradiated as in the back-reflection camera, it was found that the mean value of the lattice parameter was 4·0412 kx. at 18° c., the usual value when the material is in the metastable state. The recovery curve obtained by examination of the uneven surface of the plate B should therefore be displaced parallel to the ordinate

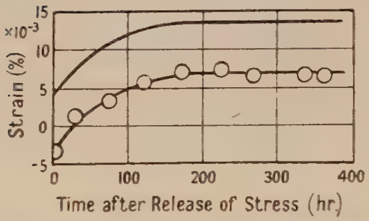

Figure 20. Recovery curve of aluminium compressed between two sheets of aluminium.

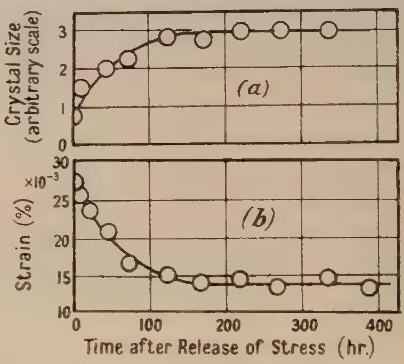

Figure 21. (*a*) Crystal growth and (*b*) recovery curve for pure aluminium after being reduced 62% in thickness in one stage.

through a distance corresponding to a percentage strain of $7{\cdot}5 \times 10^{-3}$, thus giving the upper curve in Figure 20. This experiment shows that the inside of the specimen where the crystals are allowed more freedom of movement behaves differently from the surface when this is pressed against steel plates and the crystals are constrained to move in a given direction. The same metastable state is reached both in the surface layers and in layers well within the body of the material. Work is still in progress with a view to investigating further the condition in the interior of a heavily stressed material.

## § 4. RECRYSTALLIZATION

It will be noted, in the photographs taken with a back-reflection camera (Figures 1 to 8) when the specimen was kept stationary and at a constant orientation relative to the incident x-ray beam, that in the initial stages continuous diffraction rings appear which in course of time gradually become covered with spots from small individual crystals formed during recrystallization. With the present experimental arrangement it was possible to study the growth of these individual crystals. The method adopted was to measure across a spot in a given direction and to record how this dimension changed with time. This was done for many spots in different directions across them, and the mean values found at different times after the release of the stress. Some of the spots showed little change with time, others showed marked increase, but no spot decreased in size. On the average there was an increase in accordance with the upper curve shown in Figure 21 for the specimen reduced 62% in thickness. The crystals grow rapidly in size in the initial stages but the rate of growth decreases as time goes on, the final stage, when the crystals remain constant in number and in size, being reached in about the same time that the recovery curve shows the crystallites take to reach the metastable state. The recovery curve is shown in the bottom of Figure 21. From these results it would appear that the recovery phenomenon and recrystallization are different processes but are not independent of each other. Crystals grow in size and new ones are formed when, at room temperature, the percentage strains are unevenly distributed amongst the sets of crystal planes in the specimen; crystal growth is arrested and the formation of new crystals ceases when the strains have become uniformly distributed throughout the material.

During crystallization, the crystals will assume certain orientations relative to the crystallites from which they grow. When the diffraction rings show there is preferred orientation amongst the crystallites of the matrix, there is a concentration of spots in this region of the ring, that is, the crystals tend to form with the same orientation as that of the crystallites.

In the photographs some spots were observed in course of time to break up into two separate spots. This phenomenon, as judged from the photographs, occurs fairly frequently. Its cause has not yet been fully investigated.

## § 5. DISCUSSION

The results apply strictly near the centre of the face of a polycrystalline plate of pure aluminium, between 1·5 and 2·5 mm. thick and about 7 mm. across its face, compressed between two polished steel plates, the specimen being free to expand laterally at right angles to the direction of the applied stress; they apply to a depth of at least 0·1 mm. In the body of the material the conditions are different from those at the surface as is indicated by the experiment described in § 3 (iv). This problem is being further investigated; at present no complete statement can be made regarding the behaviour of a compressed sheet at its mid-plane. The results which will now be discussed refer to the surface layers only.

The stress applied in all the cases of deformation considered is sufficient to produce a high degree of preferred orientation and the fragmentation of the crystals existing initially in the annealed specimen is heavy. The numerous crystallites produced in a compression are surrounded on all sides by transition layers. The deformation of the crystallites in the initial stages after compression, at least near the surface of the specimen, is not uniform because it is found that the planes examined yield different initial percentage strains and different recovery curves according to the angle of incidence of the radiation on the surface of the specimen.

The initial percentage strain produced in the compression of any set of crystal planes depends upon the mean inclination of those planes to the direction of the applied stress. The greater the angle between the direction of stress and the normal to the reflecting planes, the smaller is the initial percentage strain for a given compression. Hence under the present conditions of experiment, the angle which the crystal plane makes with the direction of stress seems to be a governing factor in the phenomenon. The strains in the crystallites vary from a maximum value to zero in the initial stages immediately after the release of stress; during the period of recovery the strains so adjust themselves that when the metastable state is reached they are evenly distributed; the percentage strain in the crystallites is then the same in all directions, that is, the crystal lattice is uniformly expanded. A residual strain remains which was interpreted in a previous paper (Owen, Liu and Morris 1948) as being the strain produced by the maximum tangential stress that the transitional layers can support at room temperature under the given experimental conditions. The transition layers yield only to a certain extent and are able to support tangential stresses of the magnitude of the residual stress present in the material after it has reached the metastable state. This view was based entirely on experimental observation; additional information obtained in the present work supports it.

On the dislocation theory, a metal, after deformation, is assumed to consist of an assembly of crystallites elastically deformed as the consequence of the mutual interference of various active slipping combinations, joined by transition layers composed of dislocations of opposite signs (Burgers 1947). As the neighbouring crystallites have slightly different orientations there will be an excess of dislocations of one sign, the surplus of dislocations of either sign increasing as the difference in orientation increases. Some dislocations or dislocation pairs may exist also in the interior of the crystallites, each dislocation being a centre of internal stress and producing an observed effect upon the lattice. This picture is fundamentally the same as that postulated by Owen, Liu and Morris (1948), the 'plastic', amorphous or transition layers referred to by them being now regarded as layers of dislocations.

During recovery there is no change in the positions of the transition boundaries and the orientations of the crystallites remain unaltered; a reduction in the number of dislocations only occurs. The dissolution of dislocations of opposite signs and consequently the release of stress will bring about the release of strain energy extending over a 'large' distance. The process of the dissolution of dislocations is the fundamental action of recovery as considered here. This, according to the view previously expressed by Owen and his co-workers, is the yielding of the 'plastic' or transition layers.

As the amount of cold work increases, the number of dislocations of opposite signs will also increase, and when the dislocations are close together, the interaction energy is very great. This means that the heavier the cold work applied, the greater is the rate of recovery, a conclusion which is in agreement with the present experimental results. The process of the dissolution of dislocation must cease when each transition layer consists entirely of dislocations of one sign and no dislocations exist in the interior of the crystallites. At this stage the process of recovery is at an end. The dislocations left in the transition layers are mainly those of one sign, and the number depends on the relative orientations of neighbouring crystallites. If there is a high degree of preferred orientation, and the difference of orientation of the crystals is within a definite narrow range, this number may reach a steady value and will remain unchanged. It is clear therefore that recovery cannot produce a complete removal of the effect of cold work because there is always a number of dislocations left in the transition layers after the dissolution ceases. This is in agreement with the explanation previously given; it differs only in terminology.

Reference was made earlier to the relation between recovery and recrystallization, but how recrystallization occurs was not considered. In Figure 22 are shown a number of transition or slip planes, three of which a, b and c intersect at P where there is greater disturbance of the atoms than at a point such as A on one of the planes where there is no interference from another plane. There will therefore be energy gradients in different directions. The surfaces of a crystallite are under tangential stresses which are greatest in the main direction of flow, that is, approximately at right angles to the direction of stress in heavily compressed material. If energy is released within the crystallite due to the dissolution of dislocations this will start dissolution of dislocations also in the transition layers bounding the crystallites. The resulting movements bring the atoms from the transition layers into metastable equilibrium positions on the lattices of the crystallites which will grow into the transition layers. On this view the nucleus is the crystallite which grows by the addition of atoms activated by the energy gradients within the material. In some cases adjacent crystallites are in favourable relative orientation to join together when the atoms take up their positions between them. Thus the nuclei are not formed by building up a lattice of atoms from the start but by attaching atoms to existing crystallites. Many of these crystallites may be required to unite together in heavily stressed material to form a crystal of the size that would account for the spots observed in the x-ray photographs. Comparatively few crystallites may in general be at

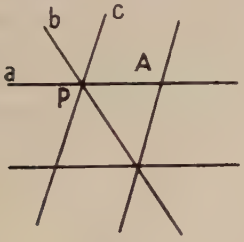

Figure 22.

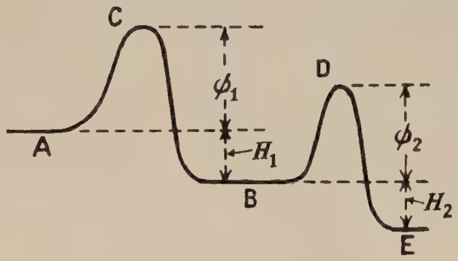

Figure 23.

the correct relative orientation with each other to unite together in this way, but those that are, grow in size during the period of recovery, and the present experimental observations show that the growth stops when the stresses become evenly distributed, or in other words the dislocations in individual transition layers all become of the same sign. This occurs when the material reaches the metastable state.

Looked at from another angle, it may be stated that when recovery takes place there is a decrease of internal energy; atoms possessing the higher potential energies move to more stable positions and attach themselves to the lattice of the crystallites present in the matrix. Suppose that the movement of atoms from the state A after plastic deformation to the strained crystallite state B (metastable state) which is of greater stability, can only occur when the energy barrier C is surmounted (Figure 23). In the case under consideration the energy is first supplied from the matrix, by the process of dissolution of dislocations. Some of the atoms in the transition layers have sufficient energy to cross the barrier and assume the strained crystalline state. If $\phi_1$ is the activation energy required to enable the atoms to move from A to B, there will be at any given instant, a certain number of atoms in the transition layers having energy greater than $\phi_1$; these will reach the state B. The activation energy $\phi_1$ will be least for atoms in the greatly disturbed regions; these will pass over the energy barrier with the least difficulty to assume the strained crystalline state. From this point of view the energy released from the matrix during recovery will be used in part to form nuclei in the sense that atoms are displaced, and become attached to crystallites which themselves act as growing nuclei. Probably a great number of potential nuclei are thus formed but do not all act as centres of recrystallization because the stress distribution in the surrounding medium is not suitable or the orientations of the neighbouring crystallites are not favourable. This second point of view leads to the same conclusions as the first.

In Figure 23 a second energy barrier is shown to represent the thermal energy required to change the strained crystalline state B (the metastable state) to the unstrained crystalline state E (the equilibrium state). The atoms are activated by strain energy to surmount the first energy barrier and by thermal energy to surmount the second. Thus for aluminium the change from the plastically deformed state to the equilibrium crystalline state takes place in two definite stages.

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# Influence of Cathode Surface Layers on Minimum Sparking Potential of Air and Hydrogen

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*ABSTRACT.* Oxide layers on the cathode produced marked irregularities in the Paschen curves for air and hydrogen, especially in the neighbourhood of the minimum $V_m$. $V_m$ in hydrogen was measured at different stages during the removal of oxide layers from the cathode, and also during the deposition of one metal on another as base. Removal of the oxide layer produced smooth Paschen curves, while a thin film of one metal on a different metal as base gave values of $V_m$ which were not characteristic of either metal in bulk. The deposition of thin metallic films had considerable effect on cathode emission owing to the alteration of the work function.

## § 1. INTRODUCTION

This paper describes an experimental study of the sparking potential $V_s$ of air and hydrogen under uniform static fields, with special reference to the influence of the cathode surface.

A study of $V_m$ in pure mercury-free hydrogen for different cathode metals has been carried out by Llewellyn Jones and Henderson (1939), who showed that there was considerable variation of $V_m$ for different metals. In general, $V_s$ appeared to increase with the effective work function of the metal, and the influence of the cathode material was most pronounced in the neighbourhood of the minimum. However, recent work in this laboratory (Llewellyn Jones 1949, Llewellyn Jones, de la Perrelle and Morgan 1951) has shown that thin oxide films on a cathode can produce enhanced pre-breakdown electron emission, and that this would be likely to affect the minimum sparking potential. It was therefore considered necessary to re-examine the effect of such layers, as well as the influence of deposited metal films, on the sparking potential, using a more severe treatment of the cathode than had previously been employed. Although values of $V_m$ of air, in particular, are of little fundamental significance owing to the complexity of the gas reactions during the passage of a discharge current, values for that gas are, nevertheless, of importance in practical applications in electrical engineering. Further, the literature of the subject shows that few determinations of $V_m$ in air have been made in gases deliberately contaminated with mercury vapour; and it is therefore of interest to investigate at the same time the role played by such contamination.

## § 2. EXPERIMENTAL PROCEDURE

Five discharge tubes were used, in which the electrodes were circular, 3 cm. in diameter with bevelled edges, mounted parallel in pairs at about 2·5 mm. apart in hard glass frames, and enclosed in hard glass cylindrical envelopes of 5 cm. diameter. Tube No. 1 had electrodes of Al and Ni, No. 2 of Staybrite steel and commercial Al, No. 3 of Cu and Staybrite steel, No. 4 of Ni and Al, and No. 5 of Ni and Staybrite steel.

The vacuum system was divided into two distinct sections, one of which was mercury-contaminated and the other mercury-free. Although the same mercury diffusion pump was, in fact, used for both sections, the exclusion of mercury vapour from the appropriate section was effected by numerous liquid air traps, by silicone-greased taps, and by only permitting movement of the gas from reservoir to discharge tubes, gauges, and then to pumps, and never in the reverse direction.

The air used was carefully filtered to remove dust, and was stored over phosphorus pentoxide. The air for the mercury-contaminated section was passed over a side-arm containing liquid mercury, and the three tubes used in this section, Nos. 3, 4 and 5, were connected together so that the gas was at the same pressure in all three. The hydrogen was generated by the electrolysis of a solution of 10% barium hydroxide in boiling distilled water, and was also dried by storage over phosphorus pentoxide. The barium hydroxide solution was heated nearly to boiling during gas generation so as to prevent dissolved gases from diffusing to the hydrogen outlet. The mecury-free gases entered the discharge tubes through liquid-air traps. Tubes No. 1 and 2 in the mercury-free section were connected together so that the gas from the reservoir entered both at the same time and at the same pressure, and they were outgassed by many hours heating at 400° C. while being evacuated. The tubes were afterwards maintained at 400° C., while containing hydrogen at a pressure of 5 mm. Hg for some hours, and were then evacuated and isolated from the vacuum system.

The required steady potential of some 1,500 V. was obtained from a storage battery and was fed to the selector switch through a resistance of 500 MΩ. Even when total currents (leakage and ionization) of as much as $10^{-9}$ amp. were passed, the voltage dropped across the resistor would have been 0·5 V.; hence for voltages around 500 V. the error due to the resistor was considerably less than 0·1%.

The definition of sparking here adopted follows the original one in this field, that is that voltage at which an extremely small current, say, of the order of or less than $10^{-10}$ amp. will flow between the plates, without the assistance of any other external ionizing agency. The procedure adopted for measuring $V_s$ was as follows. A voltage was applied to one of the tubes and the space between the electrodes was observed with the room in complete darkness; if no glow was observed, the voltage was raised in steps of two volts until a faint diffused glow in the electrode space was observed, coinciding with the onset of a small current ($\simeq 10^{-9}$ amp.) in the galvanometer. Natural ionization proved sufficient to produce adequate ionization for conveniently small statistical spark lags, and no other source was used in the later measurements.

In order that the galvanometer should record only the ionization current in the gas between the electrodes, special precautions were taken to eliminate leakage currents both over the glass surface of the tubes and in the insulation of the wires supplying the H.T. to the tubes. A specially made switch with Distrene insulation was used to distribute voltages of any given polarity to the various electrodes, and guard rings were used on all the tubes. These precautions had the effect of reducing all leakage currents below $10^{-9}$ amp. The selector switch and a reversing key made it possible to determine the sparking potentials with each of the ten electrodes in the tubes used in turn as cathode.

## § 3. THE MINIMUM SPARKING POTENTIAL OF AIR

The Paschen curves obtained are shown in Figure 1. Both with contaminated and uncontaminated air great difficulty was experienced with heavily oxidized cathodes in obtaining consistent values of the sparking potential in the neighbourhood of the minima, and in obtaining the smooth curves of the type usually published. This was because considerable difficulty was encountered in determining the actual sparking potential, or, in fact, whether there was such a critical potential at all. The galvanometer currents were intermittent, and sometimes a small steady current was observed with current surges superimposed. On viewing the tubes in complete darkness, this effect manifested itself in a diffused glow with intermittent localized glows superimposed. The result of this uncertainty was that the values of $V_s$ for a particular electrode at the same gas pressure often differed by as much as 10 volts. The curves (inset Figure 1) for contaminated air for copper cathode, for example, exhibited a pronounced second minimum at a value of *pd* of 2 mm. Hg × cm., and consistent values were not obtained.

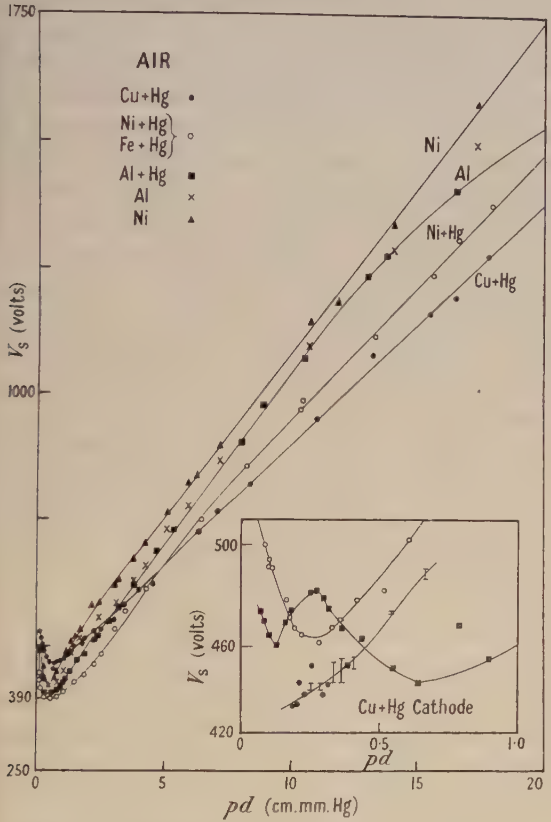

Figure 1. Paschen curves in air for various cathodes.

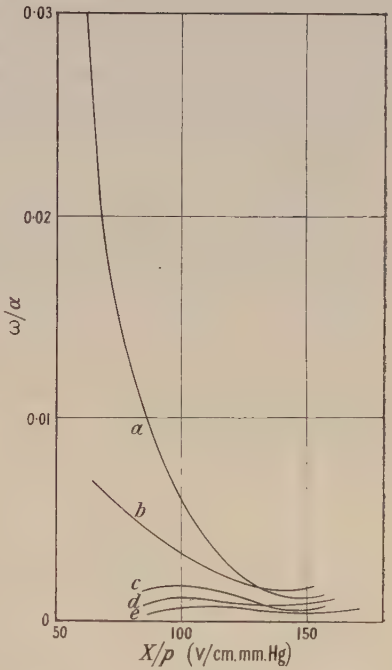

Figure 2. Variation of $\omega/\alpha$ with $X/p$ in air for cathodes of: *a*, copper amalgam, *b*, Hg film on Ni, *c*, Hg film on Al, *d*, Al, and *e*, Ni.

Previous work in this laboratory (Llewellyn Jones 1949) on the electrical properties of tungsten oxide cathode layers has shown that oxide layers on the cathode can produce greatly enhanced electron emission. Consequently, it was considered that the irregularities in the curves and the difficulty in determining precise sparking potentials were due to the changing oxygen content at the surfaces of the oxidized cathodes. These changes were, in all probability, caused by impinging positive ions from the discharge. As is shown below, this view was later confirmed when the electrodes were de-oxidized in a hydrogen glow discharge, after which smooth and reproducible curves were obtained.

It is now interesting to consider the effect of mercury vapour contamination. Figure 1 shows that values of $V_m$ in contaminated air for cathodes of oxidized

Staybrite steel, aluminium, and nickel were approximately equal (390 v.), while $V_m$ in uncontaminated air for these metals was somewhat higher. These approximately equal values of $V_m$ for steel, aluminium, and nickel in contaminated air support the view that a thin mercury layer covered those cathodes, and that the value $V_m = 390$ v. was probably that corresponding to a mercury film. However, on this view it is difficult to explain the higher values found for $V_m$ for a copper cathode in mercury-contaminated air. It is suggested, therefore, that in the case of Ni, Al, and Staybrite steel cathodes, the thin protective oxide layer normally present reduced the penetration of ions either of mercury or of the cathode metal; consequently the mercury formed a layer over the oxide. For the copper cathode, on the other hand, copper ions could penetrate its oxide layer, and thus form with the mercury an amalgam which would have a different work function from that of pure mercury. Thus traces of mercury vapour affected the minimum sparking potential by reducing, if not practically eliminating, the relative cathode effects due to use of different metals, provided an amalgam was not formed.

Calculations of the cathode secondary emission coefficient $\omega/\alpha$ for pure and mercury-contaminated air were made from the Townsend sparking relation using the values of $V_s$ and $pd$ given in Figure 1 and also values of $\alpha/p$ given by Sanders (1935) for mercury-contaminated air. (No values for pure air appear to be available in the required region of $X/p$). The results are given in Figure 2, in which curves *a*, *b* and *c* are for mercury-contaminated air, and curves *d* and *e* refer to mercury-free air for values of $X/p$ between 50 and 200 v/cm. mm. Hg. For the copper cathode covered with amalgam (curve *a*) there is evidence for a strong increase in $\omega/\alpha$ at $X/p < 100$ v/cm. mm. Hg, and also for a noticeable increase in $\omega/\alpha$ for the cases of Hg films on Ni (curve *b*) and on Al (curve *c*). No increase is exhibited for Al and Ni without Hg films (curves *d* and *e*).

## § 4. EXPERIMENTAL PROCEDURE: HYDROGEN

The irregularities in the Paschen curves obtained with air for highly oxidized cathodes have been attributed above to the effects of oxide layers on the cathode. To test this conclusion, experiments were performed with hydrogen in which the oxide layers were slowly removed from various cathodes. Paschen curves were measured at various stages during the removal of the oxide by bombardment of the cathode by positive ions of a hydrogen glow discharge between the electrodes of the tubes used. The work of Lozier (1930) and others has indicated that a fraction of the ions impinging on the cathode would then be high-speed protons. Pure hydrogen at 10 mm. Hg was admitted to the tubes and a glow discharge of 100 ma. passed, and, at first, the polarity of the electrodes was reversed at random. During this treatment, the electrodes sometimes became red-hot, and in this way it was hoped to remove the last traces of the oxide layers from the electrodes. After three hours of the above treatment the tubes were evacuated.

Subsequent to this procedure and, starting at a pressure of 45 mm. Hg, the Paschen curves for the four electrode materials were determined: the method used was exactly as described above for air. It was found, however, that except in the case of Staybrite steel sparking potentials were now easier to determine than in air. For this metal, on the other hand, the same difficulties were encountered as with air, in that the exact sparking potential could not be found. The tubes were again heated by a 100 ma. glow discharge in hydrogen for three hours, and more curves obtained; this procedure was repeated many times, and the state of each electrode, as indicated by $V_m$, for fresh specimens of pure gas carefully watched. When one particular electrode in a tube was bombarded by hydrogen positive ions continuously for some hours, the value of $V_m$ for that electrode altered steadily in one direction; but, on the contrary, when the polarity was changed every hour, the value of $V_m$ for either electrode changed at random. Moreover, visual examination of the tubes showed that sputtering had occurred. At this stage, consequently, it was decided to bombard with positive ions one particular electrode in each tube over a considerable period; and to take Paschen curves for the two electrodes in each tube at regular intervals during this treatment. In this way, about fifty Paschen curves for each of the electrodes in the tubes were determined, and the total time of bombardment of any particular electrode was approximately one hundred hours. The values of $V_s$ measured were those which corresponded to equilibrium between the hydrogen and the cathode surface.

## § 5. RESULTS: INFLUENCE OF THE DEPOSITION OF METALLIC FILMS

Typical results of these measurements are illustrated in Figure 3, which refers to tube No. 1.

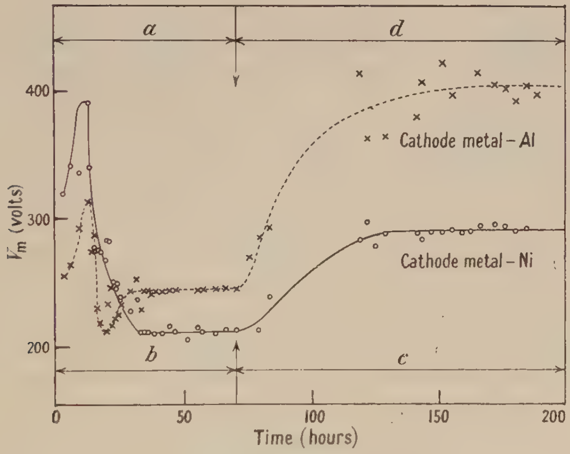

Figure 3. Variation of $V_m$ with time during: *a*, cleaning of Al cathode, *b*, deposition of Al film on Ni cathode, *c*, cleaning of Ni cathode, and *d*, deposition of Ni film on Al cathode.

During the first 20 hours of the discharge treatment, both the nickel and aluminium electrodes were bombarded by positive ions of hydrogen to remove the oxide layers from each; after this, the aluminium electrode only was bombarded for 56 hours. Under this treatment the surface of the aluminium electrode was progressively cleaned until final successive determinations of $V_m$ gave $243 \pm 3$ v. Under this bombardment treatment, however, aluminium was sputtered on to the nickel electrode, and during the first 20 hours of continuous discharge treatment, $V_m$ for the nickel cathode (now covered with a growing film of aluminium) decreased steadily from 280 v. to about 210 v. and afterwards remained constant, when successive determinations of $V_m$ gave $212 \pm 2$ v. which was 30 v. lower than $V_m$ for aluminium in bulk.

After these measurements, the glow discharge treatment was continued for a further 150 hours, this time, however, with the nickel electrode as cathode; the state of the surface of each electrode was examined as before, by measurements of $V_m$ with fresh specimens of pure gas. As a result of this treatment, it was presumed that the aluminium film on the nickel electrode was removed, as successive determinations of $V_m$ gave $289 \pm 3$ v. for a nickel cathode. During this discharge treatment nickel was sputtered on to the aluminium electrode, finally producing a clearly visible film over it. However, final values of $V_m$ for the aluminium electrode with its nickel film were not very consistent; there was considerable spread about the value of $390 \pm 20$ v., as indicated in Figure 3. It is possible that this spread was caused by the non-uniformity in the thickness of the film.

The electrodes in tube No. 2 were examined in a similar way during a total glow discharge time of 220 hours and the results obtained are given in Figure 4. After long bombardment with hydrogen positive ions, the commercial aluminium electrode gave a value for $V_m$ of $225 \pm 5$ v., while a film of that material on the Staybrite steel electrode gave the lower value of approximately 205 v. The bombarded Staybrite steel electrode gave a value for $V_m$ of about 274 v., while the film of that material on the aluminium electrode gave indecisive values between 270 and 290 v. It will be noted that the values of $V_m$ for the cathodes of Al, Ni, Staybrite steel, and commercial Al, were clearly lower than those previously given by Llewellyn Jones and Henderson (1939) for those metals when they had been degassed by heat treatment *in vacuo* but had not been subjected to the hydrogen glow discharge treatment. These lower values could be accounted for by the presence of a thin layer of positive ions of hydrogen which lowered the work function of the electrode surface by the formation of an electrical double layer (de Boer 1935).

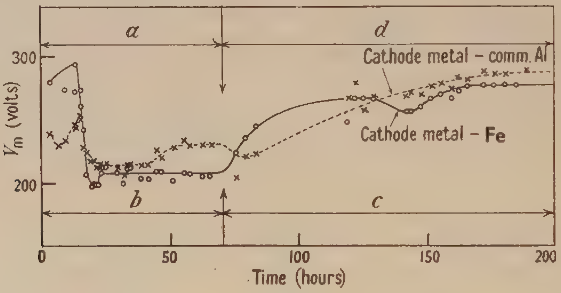

Figure 4. Variation of $V_m$ with time during: *a*, cleaning of commercial Al cathode, *b*, deposition of Al film on steel cathode, *c*, cleaning of steel cathode, and *d*, deposition of Fe film on commercial Al cathode.

The present experiments clearly demonstrate the great dependence of $V_m$ on the nature of the cathode surface. Prolonged heating by external furnaces of the electrodes at 400° c. either *in vacuo* or in hydrogen appeared to be ineffective in removing the last traces of oxide on cathode surfaces; on the other hand, bombardment of the cathode by positive ions of hydrogen (including some protons) removed oxide films in a comparatively short time. When the oxide film had been removed the onset of sparking was easily determined, and regular and reproducible Paschen curves were obtained.

A thin film of an alien metal deposited on a different metal cathode as base altered $V_m$, but the $V_m$ obtained was characteristic neither of the underlying metal nor of the deposited metal in bulk. These results do not therefore support the assumption of Jacobs and la Rocque (1947, 1948) who measured $V_m$ in the inert gases for films of aluminium, barium and magnesium deposited on a nickel cathode, and considered these values to be characteristic of cathodes of Al, Ba and Mg in bulk. Thus, in determining the value of $V_m$ (or the cathode emission) characteristic of a pure metal it is desirable to have both electrodes made of the same metal, otherwise the surface of each electrode is liable to become contaminated by deposition of atoms due to sputtering from the other electrode during the outgassing process.

One of the more striking effects of the deposition of ions of one metal upon another was the very rapid change produced of $V_m$; as shown in Figures 3 and 4 it is reasonable to regard this as being caused by a rapid change in the cathode emission $\omega/\alpha$ with the deposition of ions of the different metal. For comparison, values of $V_m$ for various cathode surfaces are given in the Table, together with the corresponding values of $X/p$ and $\omega/\alpha$; these represent average values taken over a period of some weeks. The only available values of $\alpha$ in air when $X/p > 500$ are those of Townsend (1915), and these are used for the estimation of $\omega/\alpha$; for hydrogen, Hale's (1939, 1940) values of $\alpha$ were used.

Effect of Nature of Cathode Surface on Minimum Sparking Potential

| Gas | Cathode | $V_m$ | $X/p$ | $\omega/\alpha$ |
|---|---|---|---|---|
| Air+Hg vapour | Copper amalgam | 460 | 720 | 0·004 |
| | Hg film on aluminium | 390 | 885 | 0·014 |
| | Hg film on nickel | 390 | 885 | 0·014 |
| | Hg film on Staybrite steel | 390 | 585 | 0·006 |
| Air | Oxidized aluminium | 416 | 905 | 0·01 |
| | Oxidized nickel | 421 | 957 | 0·01 |
| Hydrogen (surfaces treated by glow discharge) | Aluminium | 243 | 200 | 0·1 |
| | Aluminium deposited on nickel | 212 | 200 | 0·15 |
| | Nickel | 289 | 180 | 0·075 |
| | Nickel deposited on aluminium | 390 | 245 | 0·015 |
| | Commercial aluminium | 225 | 200 | 0·125 |
| | Aluminium on Staybrite steel | 205 | 210 | 0·15 |
| | Staybrite steel | 274 | 190 | 0·075 |
| | Steel deposited on aluminium | 282 | 190 | 0·075 |

It can be seen that very great variations ($\simeq 80\%$) in $V_m$ can be produced by changes in the cathode surface, and that these variations are greater than those previously recorded. The fact that such changes can be brought about by deposition of thin metallic films points to the modification of the effective work function of the surface by these films. The deposition of a film of aluminium either on nickel or on Staybrite steel doubled the electron emission, while the deposition of a film of nickel on pure aluminium reduced the emission by 85%, and a film of iron on commercial aluminium reduced the emission by 50%.

A further point of interest, as indicating the influence of thin surface films on the work function, is the initial increase in $V_m$ on cleaning oxidized electrodes in a hydrogen glow discharge, as is indicated, for example, by Figure 3. It is known that a monomolecular oxide film can raise the work function by setting up a negative surface charge (de Boer 1935) which repels electrons back into the metal. It is probable, therefore, that the initial increase in sparking potential (decrease of cathode emission) was due to the progressive thinning of the oxide layer, as the result of bombardment by hydrogen ions. When the film was thick, the electron emission could be comparatively high (Llewellyn Jones 1949), but when the film became monomolecular, and the work function consequently maximum (de Boer 1935) the emission would be a minimum and the sparking potential therefore a maximum. It is probable also that the effects of the deposition of metallic films could also be explained by the setting up of positive or negative ion layers on the electrode surface, but the phenomena require fuller study, and such work is in progress in this laboratory.

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# Space-Charge Distribution, Characteristic, and Current Fluctuations in 'Double Diodes'

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*MS. received* 14*th November* 1950

*ABSTRACT*. The distribution of space charge and potential in thermionic tubes with two hot electrodes ('double diodes') is derived by the method of statistical mechanics for plane parallel electrodes, first for tubes in true thermodynamic equilibrium, and then, approximately, for tubes with applied external voltage or with electrodes at slightly different temperatures. The dependence of current on voltage in the first case (characteristic), and on temperature difference in the second case is calculated. Finally a general expression for the current fluctuations ('noise') in double diodes is given and discussed. The application of this formula to other types of conductors is also indicated.

## § 1

THERMIONIC tubes with two hot electrodes do not seem to have much practical importance, but fictitious constructions of this kind have frequently been used in theoretical considerations on the question as to whether the mechanism of spontaneous fluctuations of current in such tubes is identical with that in ordinary resistors or not (Schottky 1937, North, Harris and Thomson 1941, 1942). More recently (Fürth 1947, 1948) the present author has given a general theory of spontaneous electrical fluctuations, covering both types of conductors, and experiments on 'noise' in 'double diodes' (MacDonald

1947, Knol and Diemer 1949, 1950) have actually been carried out. It therefore seems desirable to develop a theory of the space-charge distribution and the characteristic of such tubes, and to derive expressions for the current fluctuations on this basis.

§ 2

The simplest way of dealing with the space charge distribution in an electronic tube in thermodynamic equilibrium is to use the methods of statistical mechanics. This was first done in a classical paper by von Laue (1918) and more recently the method was used successfully by the author and MacDonald (MacDonald and Fürth 1947, Fürth and MacDonald 1950).

Call $\rho$ the (absolute) space-charge density and $v$ the potential, which are both functions of space; $\rho_0$ is the value of $\rho$ at a point where $v=0$. Write for brevity

$$b \equiv \boldsymbol{e}/\boldsymbol{k}T, \qquad \ldots\ldots(1)$$

where $\boldsymbol{e}$ is the magnitude of the electronic charge, $\boldsymbol{k}$ Boltzmann's constant, and $T$ the absolute temperature.

It follows from statistical thermodynamics that in statistical equilibrium

$$\rho = \rho_0 e^{bv}. \qquad \ldots\ldots(2)$$

If this is combined with Poisson's equation

$$\nabla^2 v = 4\pi\rho \qquad \ldots\ldots(3)$$

the differential equation

$$\nabla^2 v = 4\pi\rho_0 e^{bv} \qquad \ldots\ldots(4)$$

is obtained from which $v(x, y, z)$ can be determined for given boundary conditions. From this further follows $\rho(x, y, z)$ by means of

$$v = b^{-1} \log (\rho/\rho_0). \qquad \ldots\ldots(5)$$

If $v$ depends on one coordinate only (one dimensional problem) the general solution of (4) can be shown to be (Laue 1918)

$$\rho = (C^2 b/8\pi) \operatorname{cosec}^2 [\tfrac{1}{2}Cb(x+X)], \qquad \ldots\ldots(6)$$

where $C$ and $X$ are constants to be determined by the boundary conditions.

Assume that the two plane electrodes are both kept at the same temperature $T$ and situated at $x=0$ and $x=d$, and that they are both at zero potential. The boundary conditions are thus

$$v=0, \quad \rho=\rho_0 \quad \text{for } x=0 \text{ and } x=d. \qquad \ldots\ldots(7)$$

The minimum value $\rho_{\mathrm{m}}$ of $\rho$ is according to (6)

$$\rho_{\mathrm{m}} = C^2 b/8\pi, \qquad \ldots\ldots(8)$$

which for reasons of symmetry must be situated at $x=d/2$. Hence

$$\rho_{\mathrm{m}}/\rho_0 = \sin^2 \tfrac{1}{2}CbX. \qquad \ldots\ldots(9)$$

As this is certainly a small quantity, it follows approximately from (8) and (9) that

$$\frac{\rho_{\mathrm{m}}}{\rho_0} = \frac{C^2 b}{8\pi\rho_0} \simeq \left(\frac{CbX}{2}\right)^2, \qquad \ldots\ldots(10)$$

whence

$$X \simeq (2\pi b\rho_0)^{-1/2}. \qquad \ldots\ldots(11)$$

It will be shown later that $X/d$ is always a small quantity so that from (6) follows

$$Cb(\tfrac{1}{2}d+X) = \pi. \qquad \ldots\ldots(12)$$

From (6) and (12) follows

$$\rho = \frac{\pi}{2b(d+2X)^2} \operatorname{cosec}^2 \left\{\frac{\pi}{d+2X}(x+X)\right\},$$

which is evidently symmetrical about $x=d/2$ and satisfies the boundary conditions (7) very approximately. Finally, neglecting $X$ compared with $d$ and substituting the value of $X$ from (11) we obtain the final solution

$$\rho=\frac{\pi}{2bd^2}\operatorname{cosec}^2\left\{\frac{\pi}{d}\left[x+(2\pi b\rho_0)^{-1/2}\right]\right\}. \qquad \text{......(13)}$$

From (13), or directly from (8), the minimum value $\rho_m$ at $x=d/2$ is $\rho_m=\pi/2bd^2$. Thus the assumption that $\rho_m/\rho_0 \ll 1$ amounts to the condition that $\rho_0 bd^2 \gg 1$.

The potential distribution $v(x)$ in the inter-electrode space follows from (13) by means of (5). It has a minimum $v=v_m$ at $x=d/2$ of magnitude

$$v_m=b^{-1}\log(\rho_m/\rho_0)=b^{-1}\log(\pi/2b\rho_0 d^2) \qquad \text{......(14)}$$

which acts as a 'potential barrier' for the electrons trying to cross over from one electrode to the other†.

§ 3

Suppose now that a constant 'anode voltage' $v_a$ is applied to the tube so that instead of (7) the boundary conditions are now

$$v=0,\ \rho=\rho_0 \text{ at } x=0; \quad v=v_a>0,\ \rho=\rho_0 \text{ at } x=d. \qquad \text{......(15)}$$

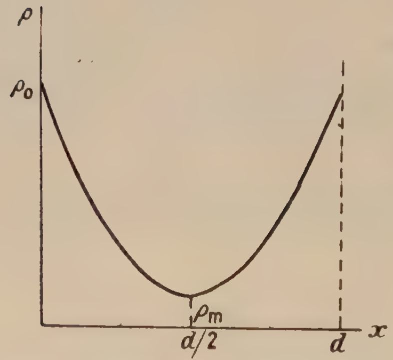

Figure 1. Space-charge distribution in a double diode.

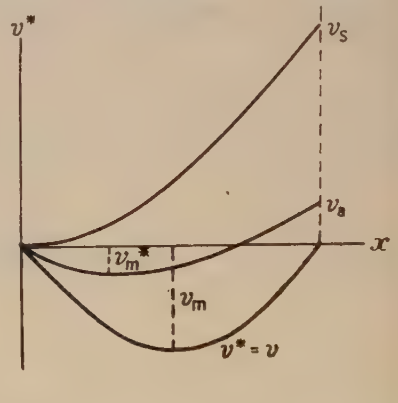

Figure 2. Potential distribution in a double diode.

This will destroy the symmetry of the potential distribution and cause the minimum of $v$ to be displaced to a smaller $x$-value (Figure 2). It is true that under these conditions Laue's method is no longer strictly applicable, as a steady current will now flow through the tube, a process which is not reconcilable with thermodynamic equilibrium amongst the electrons in the space charge. However, it seems not unreasonable to try to get an approximate solution of this problem by assuming that the space-charge distribution remains essentially unaltered by the application of the anode voltage, and that only a uniform current density is superposed over it. It was indeed shown by the author (MacDonald and Fürth 1947, Fürth and MacDonald 1950) that very nearly correct results can be obtained in this way in some similar cases.

Let us therefore assume that the space-charge distribution is still given by (13) but that the distribution of the potential $v^*$ is simply obtained by superposing the uniform field $v_a x/d$, in the absence of space charge, over the distribution (5), due to the space charge in the absence of an applied voltage. This is consistent with the condition (3), as the uniform field does not contribute to the left-hand side of (3). We thus obtain

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† The same problem has recently been treated by Knol and Diemer (1950) by a kinetic method which requires a much more complicated mathematical analysis. The results are identical with those given above. The authors also give graphs of the distribution $v(x)$ and $\rho(x)$.

$$v^* = \frac{v_a x}{d} + v = \frac{v_a x}{d} + \frac{1}{b}\log(\rho/\rho_0); \qquad \ldots\ldots(16)$$

this satisfies the boundary conditions (15), but, of course, no longer satisfies the relation (2) which only holds for true statistical equilibrium.

The position $x=\xi$ of the minimum $v_m^*$ of $v^*$ is now obtained from (13) and (16) by

$$\left(\frac{dv^*}{dx}\right)_{x=\xi} = \frac{v_a}{d} - \frac{2\pi}{bd}\cot\left\{\frac{\pi}{d}[\xi + (2\pi b\rho_0)^{-1/2}]\right\} = 0, \qquad \ldots\ldots(17)$$

whence

$$\xi = (d/\pi)\cot^{-1}(bv_a/2\pi) - (2\pi b\rho_0)^{-1/2}. \qquad \ldots\ldots(18)$$

This is indeed very nearly equal to $d/2$ for $v_a=0$, and it is smaller than $d/2$ for $v_a>0$.

The value of $v_m^*$ can now be calculated by substituting from (18) into (16). For small values of $v_a$ one has from (18) $\xi = \frac{1}{2}d - bdv_a/2\pi^2$ ($v_a$ small) and thus from (13)

$$\rho_m \simeq \frac{\pi}{2bd^2}\operatorname{cosec}^2\left(\frac{\pi\xi}{d}\right) \simeq \frac{\pi}{2bd^2}\left[1 + \left(\frac{bv_a}{2\pi}\right)^2\right]; \qquad \ldots\ldots(19)$$

finally with the help of (14) and (16)

$$v_m^* \simeq \frac{v_a}{2} - \frac{b}{2\pi^2}v_a^2 + \frac{1}{b}\log\left(\frac{\pi}{2b\rho_0 d^2}\right) + \frac{b}{4\pi^2}v_a^2 = v_m + \frac{v_a}{2} - \frac{bv_a^2}{4\pi^2}. \qquad \ldots\ldots(20)$$

As $v_m<0$, $|v_m^*|$ is seen to be smaller than $|v_m|$, i.e. the potential barrier in the inter-electrode space is lowered and at the same time shifted towards the cathode as the external voltage is increased (Figure 2).

The electronic current across the barrier is composed of two streams of electrons, one of magnitude $I_1$ moving from the cathode to the anode and one of magnitude $I_2$ moving in the opposite direction. As the former have to overcome a potential barrier of height $v_m^*$ and the latter a barrier of height $(v_m^* - v_a)$ one obtains from a well-known relation

$$I_1 = I_s \exp(bv_m^*); \quad I_2 = I_s \exp\{b(v_m^* - v_a)\}, \qquad \ldots\ldots(21)$$

where $I_s$ the 'saturation current' is given by the classical Richardson formula

$$I_s = \rho_0 a(\mathbf{e}/2\pi\mathbf{m}b)^{1/2}, \qquad \ldots\ldots(22)$$

$a$ being the electrode area.

The total current is

$$I = I_1 - I_2 = I_s \exp(bv_m^*) . \{1 - \exp(-bv_a)\}. \qquad \ldots\ldots(23)$$

This is the equation of the characteristic of the 'double diode' for $v_a>0$. It is clear that it also holds for $v_a<0$ with reversed sign of $I$ so that the characteristic is antisymmetric in shape.

$I$ is, of course, zero for $v_a=0$. For small values of $v_a$ one obtains from (14), (20) and (23)

$$\begin{aligned} I &= I_s \exp(bv_m)[\exp(\tfrac{1}{2}bv_a) - \exp(-\tfrac{1}{2}bv_a)] . \exp(-b^2v_a^2/4\pi^2) \\ &= I_s(\pi/b\rho_0 d^2)\sinh(bv_a/2)\exp(-b^2v_a^2/4\pi^2). \end{aligned} \qquad \ldots\ldots(24)$$

Expanding this expression with respect to $v_a$ we obtain

$$I = g_0\left[v_a + \frac{b^2 v_a^3}{24\pi^2}(\pi^2 - 6)\right] \simeq g_0\left(v_a + \frac{b^2 v_a^3}{60}\right) \quad (v_a \text{ small}), \quad \ldots\ldots(25)$$

where

$$g_0 = I_s \frac{\pi}{2\rho_0 d^2} = \frac{a}{2d^2}\left(\frac{\pi e}{2mb}\right)^{1/2}. \quad \ldots\ldots(26)$$

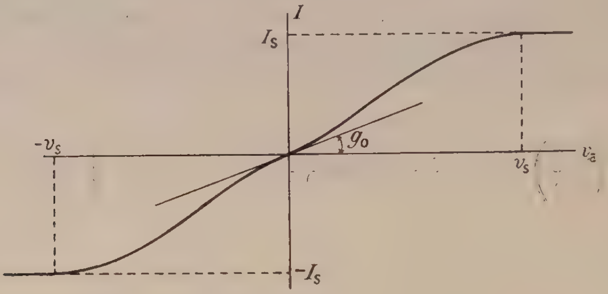

Figure 3. Characteristic of a double diode.

The differential conductivity is defined by

$$g = dI/dv_a. \quad \ldots\ldots(27)$$

Again for small values of $v_a$ one has from (25)

$$g = g_0(1 + b^2 v_a^2/20), \quad \ldots\ldots(28)$$

which becomes equal to $g_0$ for $v_a = 0$†.

The relation (26) can be used for writing the expression (11) for $X$ in the form

$$X/d = (g_0/bI_s)^{1/2}/\pi. \quad \ldots\ldots(29)$$

As the right-hand side of (29) contains only directly measurable quantities it can be used for estimating the value of $X/d$. In a typical case one has, for example, $I_s = 6 \times 10^{-4}$ amp., $g_0 = 10^{-5}\,\text{ohm}^{-1}$, $b = 5\,\text{volt}^{-1}$, which leads to $X/d \sim 0{\cdot}02$. This demonstrates that $X/d$ is indeed small, as was anticipated in §2.

One further finds from (14) and (26)

$$bv_m = \log(g_0/bI_s), \quad \ldots\ldots(30)$$

a formula which will be used later.

Formula (28) shows that $g > g_0$, i.e. that the characteristic has a negative curvature near $v_a = 0$ (see Figure 3). Under normal operating conditions this formula holds within a range of a few tenths of one volt.

For sufficiently large values of $v_a$ (23) becomes $I = I_s \exp(bv_m^*)$. This shows that the saturation value of $I$ is reached when $v_m^*$ becomes zero, and this in turn implies that $\xi = 0$ (see Figure 2). Call $v_s$ the value of $v_a$ for which the minimum of $v^*$ reaches $x = 0$ for the first time. For any anode potential $v_a$ greater than $v_s$ the current will evidently remain equal to $I_s$.

Putting the expression (18) equal to zero we obtain with the help of (11) and (29)

$$v_s = \frac{2\pi}{b}\cot\left(\frac{\pi X}{d}\right) \simeq \frac{2d}{bX} = 2\pi\left(\frac{I_s}{bg_0}\right)^{1/2}, \quad \ldots\ldots(31)$$

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† The expression (26) for the differential conductivity at zero anode voltage was also obtained by Knol and Diemer (1950).

from which $v_s$ can be calculated. In the above-mentioned typical case one obtains $v_s = 20$ volts. It has to be kept in mind, however, that the present theory becomes less and less accurate the higher the value of $I$, and that consequently formula (31) will only give the right order of magnitude.

## § 4

For tubes whose construction is not strictly symmetrical the shape of the characteristic will no longer be antisymmetric as in Figure 3, but in consequence of the second law of thermodynamics it must still pass through the origin provided the electrodes are kept at exactly the same temperature $T$. If, however, the temperatures of the electrodes are different a finite current $I_0$ will flow through the tube at $v_a = 0$ even if the tube structure is completely symmetrical. As long as the temperature difference $\Delta T$ is small compared with $T$ we may assume the space-charge distribution to remain practically unaltered except near the electrodes, and hence the derived formulae may be used for calculating $I_0$.

Assume electrode 1 to be still at temperature $T$ and electrode 2 at $T - \Delta T < T$. Equation (30) for $v_m$ will then still hold with $b = \mathbf{e}/\mathbf{k}T$ as before, but (21), in virtue of (1) and (22), has to be altered to

$$I_1 = I_s \exp(bv_m); \; I_2 = I_s(1 - \Delta T/T)^{1/2} \exp\{bv_m(1 + \Delta T/T)\}. \quad \ldots\ldots(32)$$

From (30) and (32) follows to a first approximation

$$I_0 = I_1 - I_2 \simeq I_s \exp(bv_m)(\tfrac{1}{2} - bv_m)\Delta T/T = g_0\alpha/b, \quad \ldots\ldots(33)$$

where $1/b$ has the dimension of a voltage and

$$\alpha = [\tfrac{1}{2} + \log(bI_s/g_0)]\Delta T/T \quad \ldots\ldots(34)$$

is a dimensionless quantity which is seen to be proportional to $\Delta T/T$. (In the previously mentioned typical case one has $\alpha \sim 6\Delta T/T$ and $I_0 \sim 2 \times 10^{-6}\,\alpha$ amp. $\sim 12\Delta T/T\,\mu$a.)

## § 5

So far we have only considered the properties of the *mean* current through a double diode. We now turn to the fluctuations in time of the actual current about this mean value. Current fluctuations in thermionic valves are in the main ascribed to the 'shot effect' (Schottky 1918), which is due to the fact that the current consists in the transit of more or less independently moving electrons of finite charge **e**. According to the well-known theory of this effect the mean square fluctuation $\overline{(\delta I)_f^2}$ of $I$ in the frequency range $\Delta f$ around $f$ is independent of $f$ (apart from very large $f$) and proportional to $I\Delta f$. Consequently there should be no fluctuations in a double diode with equal electrode temperatures and no applied external voltage; but this is contrary to experimental evidence.

On the other hand, the occurrence of spontaneous electric currents of a random character in closed electric circuits without a battery was first predicted by Einstein (1906) on account of the Brownian movement of the electric charge carriers, and experimentally detected by Johnson (1925)—hence sometimes called 'Johnson effect'. The mean square current $\overline{I_f^2}$ in the frequency range $\Delta f$ obeys the well-known theorem of Nyquist (1928)[*], which for a pure resistor of conductance $g = 1/R$ has the form

$$\overline{I_f^2} = 4\mathbf{k}Tg\Delta f. \quad \ldots\ldots(35)$$

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[*] A derivation of this theorem that avoids some of the difficulties inherent in its original formulation was given by the author (Fürth 1948).

If the circuit contains a battery the same phenomenon will produce fluctuations of the steady current about an average. It has been shown by Van der Velden (1947) that $\overline{(\delta I)_f^2}$ still obeys the formula (35) for ordinary metallic conductors. The 'Brownian movement of electricity' is simply superposed over the steady current, just as the Brownian movement of a heavy particle is simply superimposed over its steady downward movement, or in other words there is no 'shot' effect in such conductors. However, certain experiments on semiconductors and on metals in very thin layers (Bernamont 1934, 1937) indicate that the effect does occur in conductors of this kind.

As the relation (35) holds for any sort of conducting mechanism it was suggested by Schottky (1937) that it ought also to be valid for a double diode in thermodynamic equilibrium. In this case one has to replace $g$ by the differential conductivity $g_0$ for $v_a = 0$ so that

$$\overline{I_f^2} = 4\boldsymbol{k}Tg_0\Delta f. \qquad \ldots\ldots(36)$$

The results of recent experiments by Knol and Diemer (1949, 1950), carried out under this condition, are indeed in close agreement with this formula.

Nevertheless these authors argue that the effect observed by them is in fact due to the superposition of the shot effects of the two streams of electrons $I_1$ and $I_2$ passing through the tube in opposite directions. By Schottky's original formula each of these should produce current fluctuations

$$\overline{(\delta I_1)_f^2} = 2\boldsymbol{e}I_1\Delta f, \quad \overline{(\delta I_2)_f^2} = 2\boldsymbol{e}I_2\Delta f. \qquad \ldots\ldots(37)$$

For equal temperatures and no applied voltage one has, as shown before, $I_1 = I_2$, and as the fluctuations of $I_1$ and $I_2$ are independent of each other one gets at once from (36):

$$\overline{I_f^2} = \overline{(\delta I_1)_f^2} + \overline{(\delta I_2)_f^2} = 4\boldsymbol{e}I_1\Delta f. \qquad \ldots\ldots(38)$$

Now from (1), (21) and (30) $I_1 = I_s \exp(bv_m) = g_0/b = g_0\boldsymbol{k}T/\boldsymbol{e}$ which, substituted into (38), gives indeed (36).

But actually the relations (37) are not valid in the presence of a very considerable space charge in the inter-electrode space. The right-hand sides of these equations have therefore to be multiplied by the 'space-charge reduction factor' $\Gamma^2 < 1$, and the same factor would have to appear on the right-hand side of (37). Thus the above argument fails, and there can be no doubt that the observed fluctuation phenomenon is indeed a true 'Brownian movement of electricity'.

## § 6

Let us now assume that the mean current through the tube is not zero. A shot effect due to this current must then result which will increase the fluctuation $\overline{(\delta I)_f^2}$ above the 'equilibrium' value (36). The simplest way of dealing with this problem is to use the author's 'unified theory of electric fluctuations' (Fürth 1947, 1948) which considers 'Brownian motion of electricity' and 'shot effect' as only two aspects of one and the same phenomenon, 'thermal fluctuations'. This is in keeping with the procedure adopted in §§ 3 and 4 of the present paper, namely to apply statistical mechanics to the approximate solution of problems concerning systems that are not strictly in thermodynamic equilibrium.

The validity of equation (35) for solid metal conductors in the presence of an applied voltage $v_a$ has already been pointed out. The fluctuation magnitude is then solely determined by the conductivity $g$, equation (27). In conductors, however, where the current not only depends on a continuous potential gradient but also on discontinuous potential 'steps' (e.g. at electron emitting surfaces), one will logically expect another quantity $g_c$ to become significant which, in analogy to equation (27), is defined by $g_c = (\partial I/\partial v_c)_{v_a}$. Here $v_c$ is the sum of all the potential differences across the mentioned discontinuities. Instead of $g$, one has now to define more precisely

$$g_a = (\partial I/\partial v_a)_{v_c}. \qquad \ldots\ldots(39)$$

The author has shown (Fürth 1948) that the current fluctuations within the frequency range $\Delta f$ are given approximately by *

$$\overline{(\delta I)_f^2} = 4\boldsymbol{k}T\frac{g_a^2 + g_c^2}{g_a + g_c}\Delta f. \qquad \ldots\ldots(40)$$

Introducing the dimensionless quantity $\alpha$ by

$$\alpha = g_c/g_a, \qquad \ldots\ldots(41)$$

we can write (40) more conveniently in the form $\overline{(\delta I)_f^2} = 4\boldsymbol{k}Tg_a\Delta f(1+\alpha^2)/(1+\alpha)$. This evidently coincides with Nyquist's law (35) for $\alpha = 0$ and $\alpha = 1$ (and approximately so within these limits) but predicts increasingly larger fluctuations for increasing values of $\alpha$.

In the present case of a double diode a change $dv_c$ of $v_c$ is, on the average, equivalent to a change $dV = \frac{1}{2}dv_c$ of the value of the potential barrier $V$ at the surface of one of the electrodes and thus enters into the expression (22) for the saturation current by way of $\rho_0$ which is proportional to exp $(bV)$. Hence

$$g_c = \left(\frac{\partial I}{\partial I_s}\right)_{v_a}\frac{bI_s}{2}. \qquad \ldots\ldots(42)$$

Thus if the characteristic $I\ (I_s, v_a)$ of the tube is given, $\alpha$ can be calculated from (39), (41) and (42).

For true thermodynamic equilibrium ($\Delta T = 0$, $v_a = 0$) one has $I = 0$ and so $g_c = 0$ and $\alpha = 0$. Thus we are led back to (36). For small, finite values of $v_a$ one has from (25) and (26): $(\partial I/\partial I_s)_{v_a} = I/I_s$ and hence from (25), (28), (41) and (42)

$$\alpha \simeq \frac{Ib}{2g} = \frac{bv_a}{2}\left(1 - \frac{b^2 v_a^2}{30}\right) \quad \text{(small } v_a\text{)}, \qquad \ldots\ldots(43)$$

e.g. for $b = 5$ volt$^{-1}$, $\alpha$ becomes equal to unity for $v_a \sim 0{\cdot}5$ volt; this shows that the application of a small voltage already results in a marked increase of $\overline{(\delta I)_f^2}$ above the 'equilibrium' value.

For voltages exceeding $v_s$ one has $I = I_s$, and $g_a = 0$; hence from (1), (40) and (42) $\overline{(\delta I)_f^2} = 4\boldsymbol{k}Tg_c\Delta f = 2\boldsymbol{e}I_s\Delta f$. This is the classical Schottky formula which is known to apply in the saturation region of ordinary diodes. That it also holds for double diodes is due to the fact that no electrons emitted from the hot anode can reach the cathode under these conditions.

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* The original formula (2.40) on p. 612 of that paper contains the factor 2 instead of the factor 4 because it refers to an ordinary diode with one hot electrode. Certain criticisms raised by MacDonald (1948) against the derivation of that formula are partly irrelevant and partly the result of trivial misunderstandings (e.g. the criticism against the introduction of the quantity $g_c$). So are some critical remarks in the papers by Knol and Diemer (1949, 1950); these are to be rectified in an additional note by these authors to be published in the same journal in the near future.

Finally for $v_a = 0$ and a small relative temperature difference $\Delta T/T$ between the electrodes $\alpha$ becomes equal to the expression (34). Thus again the fluctuation magnitude will be increased over the equilibrium value. (If, as in the previously mentioned example, $\alpha \sim 6\Delta T/T$, the critical value $\alpha = 1$ will be reached for a temperature difference of about 15% between the electrodes.)

The theory was applied by the author (Fürth 1947, 1950) to fluctuation measurements on double diodes by MacDonald (1947) which showed a rapid increase of $\overline{(\delta I)_f^2}$ over the value (36) with increasing temperature above a certain critical temperature. MacDonald (1948) maintained that this effect was due to the influence of the cold glass envelope of the diode, but this explanation can hardly be correct as it would apply equally to the measurements of Knol and Diemer, whose results are in agreement with (36) within the range of temperatures used. On the other hand, it is easily seen from the formulae (43) and (34) that in the presence of a small internal E.M.F. between the electrodes or a small relative temperature difference between them the quantity $\alpha$ will first reach and later surpass the critical value 1 with increasing temperature. In any case the dependence of $\alpha$ and thus of $\overline{(\delta I)_f^2}$ on temperature can be calculated for a tube with given characteristic, without detailed knowledge of the operating conditions.

### § 7

It may finally be remarked that the formula (40) should be equally applicable to other types of conductors in quasi-thermodynamic equilibrium. It has already been mentioned that it holds for a solid metallic conductor where $g_c = 0$. A thin metallic layer, however, consists of separate grains with potential barriers at the boundaries which impede the transit of the electrons from one grain to the other. Hence, here $g_c$ is different from zero and probably increases with increasing $I$. This must result in an increase of $\alpha$ with $I$ and may provide the clue for the understanding of the results of Bernamont mentioned previously. Similarly the potential barriers at the boundaries between metals and semiconductors may be responsible for the anomalous fluctuations in circuits containing semiconductors.

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# The Correction for Dispersion of Phase Change in Fabry-Perot Interferometers

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*ABSTRACT.* A technique based on well-known interferometric procedures is described for determining the corrections for phase-change dispersion at reflection in Fabry–Perot interferometers. Results obtained during precise determinations of mercury-198 wave-engths are quoted to illustrate its application. A marked difference is found between the dispersion corrections for the silvered and aluminized mirrors actually used, and it is established that the latter have negligible corrections in the region 4047 A. to 6438 A.

## § 1. INTRODUCTION

In precise determinations of relative wavelengths with the Fabry–Perot interferometer corrections must be applied for any phase-change dispersion occurring when radiations of different wavelength are reflected at the interferometer mirrors. Light reflected at a metallic surface or film undergoes a phase change which is equivalent, in effect, to an apparent displacement of the reflecting plane with respect to the position of the metal surface. If the phase change varies with wavelength, so also does the optical distance between the mirrors on which the observed order of interference depends. Corrections for phase-change dispersion need to be evaluated with great care to attain an accuracy of ± 0·0001 A. in the measurement of relative wavelengths.

The reproducibility of wavelength measurements in terms of a spectroscopic standard is of special interest at the present time. In several laboratories interferometric investigations are in progress on the radiations of high purity emitted by single isotopes of even mass number. The objects are to assess the suitability of these radiations as reproducible length standards and to measure their wavelengths in terms of the wavelength of the red line of cadmium, which has been well established in terms of the metre. A resolution of the Ninth (1948) General Conference of Weights and Measures (Report, 1949) has envisaged the possibility of establishing the wavelength of a radiation of light, emitted by a pure even-mass isotope in specified conditions, as the ultimate standard of length.

At the National Physical Laboratory, a variable-gap Fabry–Perot interferometer is being used to measure the wavelengths of six lines in the visible spectrum emitted by the mercury isotope $^{198}_{80}$Hg from two different types of lamp. The interferometer mirrors consist of crystal quartz optical flats with high quality reflecting surfaces produced by the deposition of partially transmitting films, either of sputtered silver or of evaporated aluminium. This paper describes the technique employed to evaluate the corrections for phase-change dispersion, using a combination of the established procedures which have been previously reviewed by Meissner (1941). For silvered mirrors the results agree well with other work. Those for aluminized mirrors are interesting because they indicate that the dispersion corrections are negligible throughout the visible region from 4047 A. to 6438 A.

## § 2. METHODS OF CORRECTING FOR PHASE-CHANGE DISPERSION

Let $\lambda_R$, $N$ be the wavelength and observed order of interference for the cadmium red line and $\lambda$, $M$ the corresponding quantities for another line. The path differences are $N\lambda_R$ and $M\lambda$. Let $\phi\lambda$ represent the amount by which the path difference for $\lambda$ exceeds that for $\lambda_R$ so that

$$N\lambda_R=(M-\phi)\lambda. \qquad \ldots\ldots(1)$$

$\phi$ may have positive or negative values, depending upon the relative positions of $\lambda_R$ and $\lambda$ in the spectrum and the optical properties of the interferometer mirrors.

Fabry and Buisson (1908) used inclined (Fizeau) as well as parallel (Fabry–Perot) arrangements of the mirrors at small separations. They deduced $\phi$ from the observed orders $N$ and $M$, using sufficiently accurate values of the relative wavelengths for substitution in equation (1), re-arranged thus:

$$\phi=M-N\lambda_R/\lambda. \qquad \ldots\ldots(2)$$

Priest (1909–10) eliminated $\phi$ by means of a difference method. In this the orders were observed from two separations of the same mirrors used in the parallel condition, and the relative wavelengths were derived from the corresponding differences between the observed orders. If $N'$, $M'$ are the orders for a small separation and $N''$, $M''$ those for a large separation, then by elimination of $\phi\lambda$ in equation (1)

$$\lambda=\lambda_R(N''-N')/(M''-M'). \qquad \ldots\ldots(3)$$

Meggers (1915–16) extended the application of differences to a suitable range of separations of the Fabry–Perot mirrors, but utilized another method, described earlier by Burns, for eliminating the effects of $\phi$. In this the apparent wavelength is calculated directly from the observed orders for each mirror gap and the corrections in terms of wavelengths are derived from the systematic changes in apparent wavelength with the size of the gap.

Meissner (1941) considered that the Fizeau arrangement of the mirrors used by Fabry and Buisson is not as satisfactory as the parallel arrangement for the determination of $\phi$. With the former the correction is derived from observations of localized fringe systems and the results are liable to errors due to local irregularities in the mirror surfaces and the metallic deposits. On the other hand the results obtained with the Fabry–Perot interferometer relate to the average for the whole working aperture of the instrument.

For the determinations of $\phi$ from Fabry–Perot interferometers we employ the difference method, represented by equation (3), as a means of obtaining a first approximation to values of the mercury-198 wavelengths. By using small gaps of 5 mm. or less and large gaps of 100 mm. or more, the difference method is found to yield values of the wavelengths which are reliable to better than 1 part of $10^7$. The approximate wavelength of each mercury line is then used in equation (2) to evaluate $\phi$ from the observed orders for the smallest gap. Equation (1) may subsequently be used to calculate the values of $\lambda$ for every gap in which observations are made.

The values of $\phi$ obtained for the aluminized mirrors were subsequently checked by means of further experiments with very small gaps (0·01 mm. and 0·02 mm.) between the mirrors, which were set up for observation of the fringes of constant thickness. Tolansky (1948) has formulated the optical conditions of mirror adjustment and illumination which enable all the advantages of multiple-beam interferometry to be obtained with the wedge arrangement of mirrors.

## § 3. EXPERIMENTAL DETAILS

The N.P.L. design of Fabry–Perot interferometer consists of two adjustable mirror carriages each kinematically supported on five rounded contacts resting on the flat internal faces of a long V-shaped trough (90° included angle). Both the carriages and the trough are made of invar.

Use was made of the method due to Meggers (Meggers and Westfall 1950) for simultaneously illuminating the interferometer with light from the cadmium and mercury-198 lamps. A real image of the cadmium discharge column was suitably projected into the discharge column of the mercury lamp. As the two sources were set up in line with the optical axis of the interferometer the light from both was focused by the same condensing lens on to an aperture 2 cm. in diameter in a diaphragm placed before the mirrors, which are 5 cm. in diameter. By means of one of a suitable range of fringe projection lenses an image of the concentric ring systems in light transmitted by the mirrors was focused and centred on the slit of a prism spectrograph. Images of a diametral section of the rings in each spectrum line emitted by both sources in the range 4047 A.–6438 A. were photographed in the focal plane of the spectrograph camera lens.

Measurements of the linear diameters of five rings in each line were made from the photographic negatives in a toolmaker's microscope, and the results were used to calculate the excess fractional order at the centre of each ring system by a least-squares method (Rolt and Barrell 1929). The total orders of interference were finally derived by application of the usual Benoît–Michelson method of excess fractions. Corrections were applied to reduce the orders observed in the ambient conditions to values in dry normal air at 15° c. and 760 mm. Hg pressure*, using N.P.L. data on the refraction and dispersion of air in the visible region (Barrell and Sears 1939).

For the experiments with inclined mirrors at small separations the optical system was rearranged so that light from the cadmium and mercury lamps was condensed on an aperture 2 cm. in diameter placed at the principal focus of a lens of about 70 cm. focal length. The parallel beam emerging from this lens was directed at normal incidence on to the mirrors and an image of the fringes of constant thickness in transmitted light was projected at roughly unit magnification by a suitable lens on to the spectrograph slit.

The mirrors were assembled in a simple holder with the reflecting surfaces separated at their edges by three small pieces of aluminium foil. By means of three flexibly mounted screws pressure could be exerted on the outer faces of the mirrors for fine adjustment of the angle between the reflecting surfaces. The intersection of two cross-wires of 0·05 mm. diameter, attached to the slit face, served as a reference point for measuring the fringes. Its position was best recorded by giving two exposures, the first, with the etalon, to photograph the bright fringes crossing the slit images, and the second, without the etalon, to produce a suitable impression of the wires superposed on the fringe images.

Several spectrograms were made of the fringes located at different regions of the mirrors using foil separators of thickness 0·01 mm. and 0·02 mm. As the orders of interference were very low, Osram cadmium and natural mercury lamps of the heated cathode type were employed as sources instead of the less intense Michelson and mercury-198 lamps.

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* The conditions in which $\lambda_R = 6438{\cdot}4696$ A.

Values of the excess fractional orders at the reference point were obtained from measurements of the spectrograms in a toolmaker's microscope. The total orders of interference were then derived and used to calculate $\phi$. For this purpose the values of cadmium and natural mercury wavelengths required in equation (2) need only be known to 0·05 A. in order to yield an accuracy within 0·001 in the calculated values of $N\lambda_R/\lambda$.

## § 4. RESULTS FOR THE SILVERED MIRRORS

The silvered mirrors produced by cathodic sputtering had a reflecting power in red light of 85–90%. For the determinations of mercury-198 lines with the Fabry–Perot interferometer six mirror separations ranging from 4·7 mm. to 100·0 mm. were used. An additional observation was made with a 4·3 mm. gap after the work on the other gaps was completed. Although the mirrors exhibited some signs of tarnishing at this stage it was established that no systematic changes in $\phi$ had taken place. The corrections for dispersion of phase change are shown in Table 1, where the values of $\phi$, given by equation (2) for the mean of the 4·7 mm. and 4·3 mm. gaps, are compared with the values quoted by Williams (1948) for a typical pair of cathodically silvered mirrors.

Table 1. Values of $\phi$ for Silvered Mirrors

| $\lambda$ (A.) | N.P.L. | Williams | $\lambda$ (A.) | N.P.L. | Williams |
|---|---|---|---|---|---|
| 6500 | — | (0·000) | 5000 | — | +0·011 |
| 6438 | (0·000) | — | 4500 | — | +0·023 |
| 6000 | — | +0·002 | 4358 | +0·023 | — |
| 5791 | 0·000 | — | 4078 | +0·039 | — |
| 5770 | 0·000 | — | 4047 | +0·040 | — |
| 5500 |  | +0·005 | 4000 | — | +0·045 |
| 5461 | +0·003 | — |  |  |  |

## § 5. RESULTS FOR THE ALUMINIZED MIRRORS

The same optical flats were used after removing the silver deposits, but the aluminized reflecting surfaces were produced by the evaporation method. The reflectivity of both mirrors for the whole visible region was between 80% and 85%, the corresponding transmissions varying from 6·5% to 2·5%. Mirror separations of 1·6 mm., 2·5 mm., 5·5 mm. and extending up to 124·9 mm. were used. The values of $\phi$ were derived from the observations on the three smallest gaps after calculating the mean approximate values of wavelength from the respective differences with the 124·9 mm. gap. The results are shown in Table 2.

Table 2. Values of $\phi$ for Aluminized Mirrors

| $\lambda$ (A.)<br>Gap (mm.) | 4047 | 4078 | 4358 | 5461 | 5770 | 5791 |
|---|---|---|---|---|---|---|
| 1·6 | +0·003 | +0·001 | +0·001 | +0·002 | 0·000 | −0·002 |
| 2·5 | −0·006 | −0·005 | −0·005 | −0·003 | −0·002 | 0·000 |
| 5·5 | −0·005 | −0·005 | −0·005 | +0·001 | +0·001 | +0·002 |
| 0·01 and 0·02* | +0·001 | +0·002 | +0·001 | 0·000 | −0·001 | +0·004 |

*Additional values $\left\{\begin{array}{l}\lambda=4678\text{ A.},\ 4800\text{ A.},\ 5086\text{ A.}\\ \phi=-0{\cdot}001,\ -0{\cdot}001,\ 0{\cdot}000\end{array}\right.$

The average value of $\phi$ derived from the three gaps is negligible for the yellow and green lines and about $-0{\cdot}003$ for the three lines in the blue and violet. If this correction is applied to the results obtained from the whole series of nine gaps measured, the final wavelengths for these lines are reduced by $0{\cdot}000\,0_5$ A. relative to those obtained if $\phi$ is considered to be negligible throughout.

It was in order to settle this point that the experiments with very small separations of 0·01 mm. and 0·02 mm. were undertaken. The results of these experiments gave values of $\phi$ which are also shown in Table 2. With the evidence thus provided, the correction for phase-change dispersion at the aluminized mirrors was finally accepted to be negligible (i.e. within ± 0·002) throughout the visible spectrum.

Some confirmation of the accuracy of the corrections applied for phase-change dispersion at the two kinds of mirror is provided in Table 3, showing the final values of wavelengths obtained for the intense visible triplet $6p\,{}^3P^0_{2,1,0} - 7s\,{}^3S_1$ in mercury-198.

Table 3. Comparison of Wavelengths (in A.) ($\lambda_R = 6438{\cdot}4696$ A.)

| Line | Silvered mirrors | Aluminized mirrors |
|---|---|---|
| Green | $5460{\cdot}7527_7$ | $5460{\cdot}7528_4$ |
| Blue | $4358{\cdot}3371_3$ | $4358{\cdot}3372_1$ |
| Violet | $4046{\cdot}5711_1$ | $4046{\cdot}5712_1$ |

The source of mercury-198 lines for both series of determinations was an electrodeless lamp constructed at the National Bureau of Standards (Meggers and Westfall 1950). It was operated in the recommended conditions. The cadmium lamp used as source of the reference wavelength $\lambda_R$ was the Michelson type, constructed and operated according to the international specification.

## § 6. DISCUSSION

If the utmost precision is desired in wavelength determinations with the Fabry–Perot interferometer, it appears advisable to extend the normal series of observations at different mirror separations down to very small path differences of 50 orders or so. This enables the corrections for dispersion of phase loss to be checked and established with high fidelity. As it is generally inconvenient to use the parallel mirror arrangement for such small separations, the optical system and the mirrors must be rearranged for observation of the localized fringes of constant thickness. When high quality optical flats deposited by the evaporation method are used, and the observational conditions are suitably varied, the results obtained from the Fizeau arrangement of the mirrors appear to be very reliable. In the conditions described above, little justification has been found for the earlier mentioned criticism by Meissner (1941) of the method.

Although Meggers and Westfall (1950) originally reported results with aluminized mirrors which suggested that phase-change dispersion was observable in the visible spectrum, further work with the same mirrors by Meggers and Kessler (1950) does not indicate any appreciable dispersion between 2500 A. and 5800 A. A similar conclusion is reported by Schulz (1950) for aluminized mirrors used in white light interferometry.

Some part of the difference between the observed optical property of silvered and aluminized mirrors may be due to the different absorptions for visible light possessed by the surface film existing on the two metal deposits. The tarnishing of silver in the atmosphere may be attributed to the formation of a silver sulphide layer, which partially absorbs incident light, but the growth of the layer does not appear to influence the value of $\phi$, at least not appreciably in the early stages. It is well known that the surface of a freshly evaporated aluminium film immediately oxidizes upon exposure to the atmosphere and that the transparent aluminium oxide film thus produced increases in thickness with time, reaching a steady value between 20 A. and 40 A. after several months (Cabrera, Terrien and Hamon 1947). Here, again, no definite change in $\phi$ with time was observed during this work, although measurements were made a few days after the evaporation and at various times up to a year afterwards.

## ACKNOWLEDGMENTS

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# Measurement of the Thickness of Thin Films by Multiple-Beam Interferometry

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*ABSTRACT.* The method of measuring the thickness of thin films by multiple-beam Fizeau fringes has been studied in order to assess the accuracy attainable. In this connection the small-scale irregularities on the surfaces of optical flats and other optically worked glass have been studied by multiple-beam methods. The effect of these irregularities on the accuracy of the thickness measurements is shown and the conditions for optimum accuracy are discussed.

Thickness measurements have been made on films of silver and of lithium fluoride with several different metals as reflecting layers. The measurements cover the range 120 A. to 1500 A. The results for the different reflecting layers show agreement to within the observational error provided that measurements are made within a short time of the preparation of the films. With silver as the reflecting layer an accuracy of ±10 A. can be expected if suitable precautions are taken.

## § 1. INTRODUCTION

THE method of measuring the thickness of thin films by multiple-beam interferometry as described by Tolansky (1948, p. 147) is now well established. The film is deposited on a smooth substrate in such a way as to possess a well-defined edge, and a highly reflecting opaque layer, usually of silver, is deposited over an area which includes the edge of the film. A wedge is then formed between the silvered film and a heavily silvered optical flat and the system is examined by reflection with parallel monochromatic light. The wedge is adjusted so that the fringes of constant thickness (Fizeau fringes) cross the edge of the film at right angles. From the displacement of the fringes on crossing the step, the thickness of the film is easily deduced. This paper reports the results of an investigation on the accuracy to be expected by this method and of some factors influencing the precision attainable. Among these factors are (i) the effect on the fringe displacement of small irregularities in the surface of the supporting plate and of the optical flat and (ii) the accuracy of locating the centres of the fringes and hence the precision with which the displacements involved may be determined. Both these factors depend on the wedge angle between the test plate and the optical flat, that is, on the number of fringes per centimetre along the step. Errors due to these two causes vary in opposite directions as the wedge angle increases so that conditions for minimizing the combined error may be deduced.

The first part of the paper (§§2 and 3) is devoted to the application of a multiple-beam method to the examination of the small-scale irregularities on the surfaces of the glass plates and flats used in the thickness determinations.

A series of thickness measurements has been made on films of lithium fluoride and of silver using five different metals as reflecting layers. Two experimental arrangements were used for the measurements; their respective merits are discussed.

## § 2. EXAMINATION OF SURFACE IRREGULARITIES

The contours of the surfaces used were examined by the method employed by Tolansky and Wilcock (1946) for the examination of crystal surfaces. The surface to be investigated is heavily silvered (reflection coefficient ~90%) and is placed close to and parallel to a similarly silvered optical flat. The intensity of monochromatic light transmitted by the system depends on the separation of the surfaces and passes through a very sharp maximum for separations $n\lambda/2$, where $n$ is an integer. If the absorption in the silver film is neglected, the intensity $I$ transmitted is related to the separation $t$ by the relation

$$I = I_0/[1 + F\sin^2(2\pi t/\lambda)] \qquad \ldots\ldots(1)$$

where $I_0$ is the maximum intensity, $F = 4R/(1-R)^2$, and $R$ is the geometric mean of the reflection coefficients of the two silver layers. The variation of intensity with separation for two surfaces for which $R = 0{\cdot}90$ for the wavelength $\lambda = 5461$ A. is shown in Figure 1. (Within the range ±100 A. of the value of for which the maximum intensity occurs, the approximation $\sin 2\pi t/\lambda \simeq 2\pi t/\lambda$ is valid to within 0·5% in this case.) In the neighbourhood of half the peak intensity the variation of intensity with separation is very rapid and from the slope of the $(I, t)$ curve it is seen that a variation of separation as small as 10 A. produces an easily measurable change in the intensity transmitted. In practice, the separation is adjusted until the maximum is obtained and a photograph taken. (Due to the relatively slow variation of intensity with separation at the maximum, this setting may be made visually.) The separation is then changed so that approximately half the peak intensity is obtained and a second photograph taken with double the exposure of the first. In this way the photographic densities of the two plates are approximately the same and accurate comparison of densities is possible. (Separate calibrations of each plate are made.) The variation of photographic density across any desired line of the plate is measured using a microphotometer. Variations of intensity are deduced from which the variation in separation of the two surfaces is obtained. The method yields the *difference* in the contours of the two plates rather than the actual contour of either.

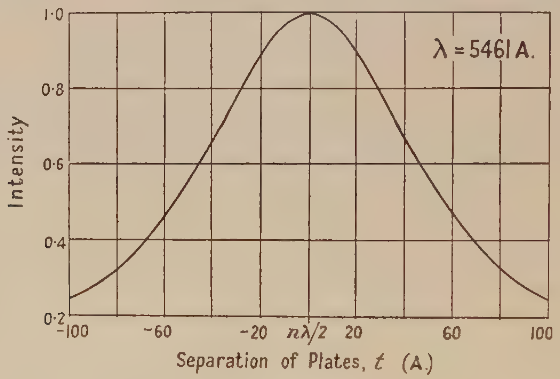

Figure 1. Variation of transmitted intensity with separation for two parallel plates with reflection coefficients 0·90.

## § 3. TESTS MADE ON THE OPTICAL SURFACES

The glass plates with which the subsequent thickness measurements were made were optical windows 2·2 cm. × 1·1 cm. over which area they were flat to within one or two wavelengths. The optical flat used was one of a pair for which the makers guaranteed flatness to within $\lambda/20$ over a diameter of 6 cm.

Preliminary tests were made by matching the two optical flats one against the other and investigating the surface irregularities as described in §2. A magnification of 5·5 was used in making the Fizeau photographs and each record covered a section of the flats 4 mm. in diameter. The resolving power of the microphotometer enabled irregularities over distances along the surfaces of the order 30 microns to be detected. The overall magnification was approximately the same as that used for the subsequent thickness measurements. Thus irregularities too small to be detected by this lateral magnification are likewise too small to introduce errors in the thickness determinations. At least two records were taken at each position on the flats, the separation being adjusted to give intensities slightly over and slightly under the half-maximum. In this way, regions for which the sensitivity is low on one photograph fall on the high-sensitivity part of the second record.

The use of the Airy summation for the case of two plates with irregularities in the surface is justified (*a*) provided that the plates are placed very close together and (*b*) so long as the residual angles between the two local surfaces, due to the irregularities, are not too large. These limitations have been investigated by Brossel (1947) who has derived expressions for the intensity distribution for a wedge. For separations between the surfaces of the order used in these experiments ($<0{\cdot}01$ mm.) and for residual angles of the order found for these surfaces ($<10^{-5}$ radian) the more exact expressions for the intensity distribution reduce to the Airy form within very close limits.

A typical result for the tests on the two optical flats is shown in Figure 2(*a*). When the very high resolution involved is considered, the agreement between the two sets of figures is satisfactory. The plates appear to be flat locally to within very much less than the guaranteed $\lambda/20$. The surface contours of a number of the optical windows were examined by this method and a typical result is shown in Figure 2(*b*). As might be expected, this shows larger irregularities than those of the optical flats although in no case did the height of local irregularities above the mean surface exceed $\lambda/150$. Irregularities of this order are deduced, for optically worked surfaces, from measurements using fringes of equal chromatic order by Scott, McLauchlan and Sennett (1950).

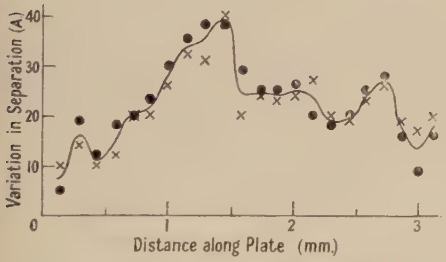

Figure 2 (*a*). Surface irregularities. Two Zeiss optical flats.

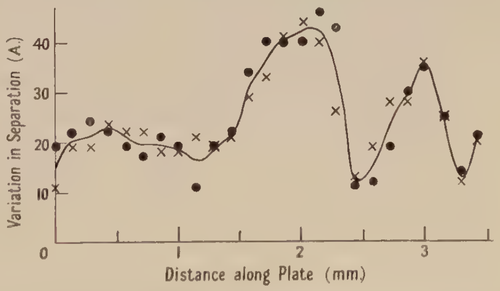

Figure 2 (*b*). Surface irregularities. Optical window glass matched against a Zeiss flat.

## §4. THE INFLUENCE OF SURFACE IRREGULARITIES ON THICKNESS MEASUREMENT

The Fizeau fringes used for thickness determination occur at places at which the separation between the silvered surfaces is $n\lambda/2$ (neglecting a very small correction due to the lack of exact phase agreement of the multiply reflected beams). Irregularities in the surfaces will thus give rise to a lateral displacement of the fringes and hence cause uncertainty in the thicknesses obtained. The extent of the error introduced will depend on the scale of the irregularities relative to the fringe displacements involved and also on the accuracy with which the positions of the fringes can be located. The latter factor depends on the wedge angle used in the thickness measurement and on the fringe width. A small wedge angle results in widely spaced fringes and the fringe displacements can be accurately measured. The errors arising from displacements of the fringes due to undulations in the surfaces of the type shown in Figure 2 increase with decreasing wedge angle. There are also fewer steps to measure for a given length of film edge. For any pair of surfaces with given imperfections, an optimum value of fringe spacing therefore exists. This value depends on the thickness of film being measured. For the surfaces used in these experiments the optimum value ranges from about 10 fringes per centimetre for films 100 A. thick to about 50 fringes per centimetre for films of 2000 A., assuming an accuracy of 5 $\mu$ in the displacement measurement. This is justified when silver or aluminium are used as reflecting layers; only for metals of high reflectivity are the Fizeau fringes sufficiently fine for this precision to be attained.

## § 5. THICKNESS MEASUREMENTS USING DIFFERENT METALS AS REFLECTING LAYERS

### (i) *Preparation of Test Specimens*

The optical windows on which the test films were to be deposited, after a preliminary examination for approximate flatness against an unsilvered optical flat, were cleaned by the process suggested by Colbert and Weinrich (1945). After chemical cleaning, a thin layer of lanoline is applied and then removed by successive applications of French chalk on virgin felt. The plates should show no breath figure after this treatment. A glow discharge at a pressure of 100 microns for 10 minutes completes the process.

In order that the displacement of the fringes at the edge of the test film can be easily measured, it is essential that the edges of the film be well defined. The plate was therefore supported on two inclined razor blades so that the extreme edges of the blades were in contact with the plate.

A strip of the test film 4 mm. wide was evaporated down the middle of the plate from a source 8 mm. × 5 mm. at a distance of 40 cm. With this arrangement the thickness of the deposit can be expected to be uniform to within 0·05%.

An opaque layer of silver was next deposited over the whole of one edge of the test film. Measurements along this step confirmed the constancy of thickness of the film. The other metals used as reflecting surfaces were Al, Ni, Cr and Sn and these were deposited as small squares on the other edge of the test film as shown in Figure 3. The target was shielded from the source during the first few seconds of the evaporation so as to minimize contamination of the film by impurities on the surface of the source (due, for example, to atmospheric oxidation). The rate of deposition of the silver and aluminium films was approximately 20 A. per second; that for nickel, chromium and tin was of the order 5 A. per second. The pressure in the system, which was pumped by an oil pump without trapping, was never allowed to rise above $10^{-5}$ mm. Hg as measured by a Penning gauge situated inside the evaporation chamber. Measurements were made on the films as soon as possible after preparation and in all cases on the same day.

### (ii) *Measurements on the Fringes*

The usual optical arrangement for illuminating the wedge system with parallel monochromatic light and viewing the reflected system at normal incidence was employed. The optical flat was silvered to give a reflectivity of about 90%. The test plate and optical flat were mounted on a device for producing the necessary fine control of separation and tilt of the two plates; this has been described by the author elsewhere (Heavens 1950). The fringes were viewed with a low-power (×55) microscope.

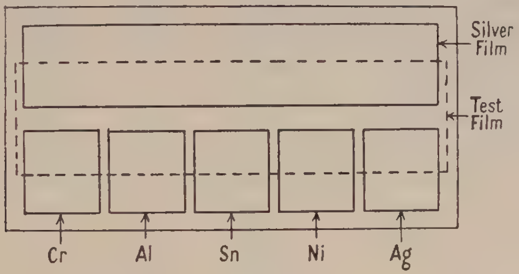

Figure 3. Arrangement of reflecting deposits on test specimens.

Two methods were employed for making the actual fringe measurements. In the first method, readings were taken directly with a micrometer eyepiece. In the second, the fringes were photographed using the objective alone (×5·5) and the photographs measured using a travelling microscope. The micrometer eyepiece gave readings to 10 microns while the travelling microscope could be relied upon to within 2 microns. In the case of fringe systems with Ni, Cr or Sn as the reflecting layers the fringes were so broad that the photographic method offered no advantage. While the photographic method involves additional processes, its chief advantage lies in the fact that, with suitable attention to the optical components the time of exposure may be made very short and effects of slow drifts (due, for example, to temperature changes) are eliminated. The use of a bloomed collimating lens and a 45° reflecting plate coated with a low-index–high-index quarter-wave pair to give enhanced reflection at an angle of incidence of 45° for mercury green light enabled the exposure time to be reduced to 1 second. This type of selective reflector has been described by Turner (1950).

### (iii) *Results of the Measurements*

The results of the thickness measurements are collected in Tables 1 and 2; PH indicates measurements made on the fringe photographs, ME shows the micrometer eyepiece figures. It was observed that the internal consistency of the step measurements was better for the LiF films than for the Ag films. This difference is probably due to the nature of the films themselves. Electron microscope observations and refractive index determinations on evaporated films of lithium fluoride (Schultz 1949) suggest that the interiors of the films are porous but that the porosities are not accessible from outside the film. Working with thick films (>20,000 A.) Schultz finds that such films contain both accessible and inaccessible voids but that the former type is absent for films less than 20,000 A. thick. He finds that such thin layers have only some 70% of the bulk density but that the layer does nevertheless behave

Table 1. Test Films of Silver

| Specimen | | Thickness (A.) for reflecting layers of | | | | |
|---|---|---|---|---|---|---|
| No. | Method | Ag | Al | Sn | Ni | Cr |
| 1 | PH | 118±10 | 133±12 | — | 130±30 | — |
| 2 | PH | 325 | 335 | — | 320 | — |
| | ME | 339 | 338 | — | 340 | — |
| 3 | PH | 413 | 402 | — | 420 | 400 |
| | ME | 401 | 420 | — | 390 | 430 |
| 4 | PH | 630 | 647 | 620 | 600 | 630 |
| | ME | 639 | 631 | 640 | 640 | 620 |
| 5 | PH | 763 | 781 | — | 750 | 730 |
| 6 | PH | 822 | 815 | — | 840 | 830 |
| 7 | PH | 922 | 890 | — | 890 | 900 |
| | ME | 910 | 901 | — | 930 | 880 |
| 8 | PH | 1085 | 1085 | — | 1120 | 1060 |
| | ME | 1070 | 1098 | 1070 | 1140 | 1100 |
| 9 | ME | 1082 | 1097 | — | 1040 | 1040 |
| 10 | PH | 1270±16 | 1290±20 | — | 1320±60 | 1230±60 |

Table 2. Test Films of Lithium Fluoride

| Specimen | | Thickness (A.) for reflecting layers of | | | | |
|---|---|---|---|---|---|---|
| No. | Method | Ag | Al | Sn | Ni | Cr |
| 11 | PH | 170 | 160 | 164 | 150 | 160 |
| 12 | PH | 220 | 225 | — | 200 | — |
| | ME | 227 | 216 | — | 220 | 240 |
| 13 | PH | 273 | 271 | — | 280 | 270 |
| | ME | 282 | 274 | — | 260 | 280 |
| 14 | PH | 471 | 460 | 479 | 450 | 490 |

as a continuous film due to the inaccessibility of the porosities. Working with thin films of silver, Sennett and Scott (1950) have shown that the structure is certainly not continuous for thicknesses less than 200 A. for a rapidly evaporated film. There is thus the possibility of filling in the gaps in such films by the added reflecting layer. The consistency of the figures for films thicker than about 200 A. with the different reflecting layers, together with the electron microscope evidence of the structure suggests that such filling-in does not occur for these films. The concept of thickness of a very granular film is not well defined; the mass per unit area is a more useful datum.

As is to be expected on account of their lower reflectivities, the results for the Sn, Ni and Cr layers show a larger spread. There appears to be no marked trend in these results, suggesting that metals other than silver may be employed for thickness measurement in cases where the use of silver is undesirable. The large discrepancies reported by Avery (1949) with chromium and silver were not confirmed. Certain anomalies with these two metals were, however, observed and are discussed below.

### (iv) *The Effect of Ageing*

It is important to know whether any change takes place in the composite film, after the deposition of the reflecting layer, of such a nature as to give erroneous results if measurements are not made immediately after silvering. Further measurements were made on certain specimens over a period of nine weeks.

Difficulty was experienced with the chromium step on some specimens as the presence of chromium seems to accelerate the tarnishing action of the atmosphere on the silver layer *below* the chromium. The fringes crossing this part of the plate became broken up when the specimens were a week old and no further measurements were possible. The results of these tests are shown in Table 3 for Specimen 9.

Table 3

| Age of specimen | Thickness (A.) from reflecting films of | | | |
|---|---|---|---|---|
| | Ag | Al | Ni | Cr |
| ½ hour | 1082 | 1097 | 1040 | 1040 |
| 5 days | 1084 | 1085 | 1120 | 1160 |
| 12 ,, | 1075 | 1025 | 1050 | 1240 |
| 28 ,, | 1089 | 1069 | 1061 | — |
| 63 ,, | 1067 | 1081 | 1090 | — |

The accuracy of the second and third chromium figures is low due to the deterioration of the surface. The considerable oxidation does, however, show in the apparent increase in thickness. There appears to be no significant trend in the figures from the Ag, Al or Ni layers.

## § 6. CONCLUSIONS

Examination of the surface contours of optically polished glass by means of multiple-beam interference shows that the departure from the mean level due to local irregularities is not usually greater than $\pm\lambda/150$. The influence of these small-scale irregularities on the accuracy of film thickness measurements show that errors of the order 10–15 A. may arise from this cause.

Thickness measurements made with five different metals as reflecting layers give concordant results within the accuracy to be expected provided that measurements are made on freshly prepared films. Errors may arise due to the oxidation of the layer, even when covered by a reflecting layer, and may amount to as much as 15–20%.

## ACKNOWLEDGMENTS

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# LETTERS TO THE EDITOR

## Production of Recombination Spectra

In view of the uncertainty about the ways in which positive ions and electrons recombine in the gas phase (Bates 1950), it seems of interest to record an instance in which an experiment on recombination requires re-interpretation, and to call attention to the need for caution in analysis of experiments with intermittent discharges.

Freeman (1930) obtained an argon spectrum with enhancement of high members of the 2p-md and 2p-ms A I series, by passing A.C. discharges between electrodes at the ends of one arm of an elongated H-shaped tube, and examining a glow which appeared in the other arm, which had a diameter of 1 cm. He attributed the presence of highly excited atoms in the latter to recombination of positive ions and electrons which had diffused from the main discharge.

The difficulty about this explanation is that it does not take account of the probable loss of charged particles by diffusion to the wall of the side-tube. For a distance of order of a tube diameter from the main discharge conditions will be complicated. If however all ions and electrons reaching the wall recombine at the surface, it will probably be an adequate approximation at greater distances to treat the main tube as a source across which the concentration $n$ is given by

$$n=(N+n_0e^{ipt})J_0(sr/R), \qquad \ldots\ldots (1)$$

where $r$ denotes distance from the axis of the side-tube, $R$ the tube radius, and $s$ the first zero (2·40) of the Bessel function $J_0$. The second term in (1) allows for the fluctuation of the concentration about a mean value given by the first term, with the angular frequency $p$ of the discharge. Denoting axial distance from the main discharge by $z$, and neglecting the curvature of the axis of the side-tube and the fact that it is of finite length, the solution of the diffusion equation $D\nabla^2 n-\partial n/\partial t=0$ with constant diffusion coefficient $D$ is

$$n/J_0(sr/R)=Ne^{-sz/R}+n_0e^{ipt}\exp\{(\cos 2\alpha)^{-1/2}e^{i\alpha}(-sz/R)\}, \qquad \ldots\ldots (2)$$

where $\tan 2\alpha=(p/D)(R/s)^2$.

The second term in (2) represents a damped wave with the frequency $p$. If the amplitude of the wave is put proportional to $\exp(-Kz)$,

$$K=(\cos 2\alpha)^{-1/2}(1+\cos 2\alpha)^{1/2}s/(2^{1/2}R). \qquad \ldots\ldots (3)$$

$K$ is greater than $s/R$, and tends to $(p/2D)^{1/2}$ as $p$ tends to infinity and to $s/R$ as $p$ tends to zero. In the latter case (2) becomes

$$n=(N+n_0)J_0(sr/R)e^{-sz/R}. \qquad \ldots\ldots (4)$$

From Freeman's description of the discharges, $N+n_0$ was probably of order of $10^{10}$ to $10^{11}/\text{cm}^3$ so that, from (3) and (4), $n$ would have fallen to not more than 50 to $500/\text{cm}^3$ on the axis at a distance 4 cm. from the main discharge. This is incompatible with the observed intensity of the light. The effect of volume recombination will be to make the decrease of $n$ with increase in $z$ even greater. A possible explanation of the discrepancy might be that the wall was not in a state permitting of surface recombination, but this is unlikely.

Using tubes similar to Freeman's, we have found that penetration of the discharge into the side-arm is much reduced if the main discharge is passed from a high-tension battery. With this, the lateral discharge was visible for less than 1 cm. It was remarkable, however, that when the battery was replaced by an 800 v. nominal D.C. source, with a mean ripple of less than 3%, obtained by transforming, rectifying and smoothing the 50 c/s. 220 v. A.C. mains supply, the long lateral discharge reappeared. As shown by (3), this is unlikely to have been due to diffusion having been increased by the ripple.

It seems probable that Freeman was observing the spectrum of a species of A.C. discharge passing from the main discharge to the air through the length of the side-arm. It cannot be decided without more elaborate investigation if the lateral discharge in his and our experiments was taking place at the frequency of the main discharge, or if, as is more likely, it was due to voltage impulses of higher frequency generated by the action of the fundamental

A.C. component of the power supply on the ionized gas in the main discharge. It is also possible that the lateral discharge was of subnormal type, without a fully developed plasma and wall sheaths, although presumably a recombination spectrum can be produced in the off-phase of this as well as in the off-phase of a normal discharge.

We are indebted to Mr. D. R. Bates for comments on this work.

Physics Department,
Queen's University, Belfast.
2nd March 1951.

K. G. Emeléus.
G. A. Smith.

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## The Background Counting Rate in a Geiger–Müller Counter

The background counting rate in a Geiger–Müller counter arises partly from cosmic rays and partly from local natural activity. Gamma-radiation from the latter may be attributed to the pervading uranium, thorium and potassium content of the earth's surface, to atmospheric activity due to the emanations diffusing from the surface and to active contamination of apparatus, including the Geiger counter. Quantitative assessment of these effects and of their contribution to the Geiger background counting rate assists in establishing limits of detectability in prospecting.

During a radioactivity survey of S.W. England and N. Wales during the summer of 1950, the counting rate due to gamma-ray background was measured in various localities and related to radiometric assays of soil activity. The apparatus used consisted of a probe unit containing three Geiger counters type CV2148, enclosed in a cylindrical dural case of wall thickness 1·25 mm., and this was coupled to a cold cathode valve ratemeter circuit registering on a 0–50 microammeter. The Geiger counters, connected individually so that simultaneous pulses are not lost in coincidence, had cylindrical graphite cathodes of length 20 cm. and diameter 2·1 cm. coated inside glass envelopes of wall thickness 0·6 mm. The tubes were filled with a mixture of argon and ethyl bromide at pressures of 9 cm. and 1 cm. respectively.

| Sample | Ratemeter reading | Radiometric assay | | | Spectrographic analysis |
|---|---|---|---|---|---|
| | Specific count rate | % $U_3O_8$ | % $ThO_2$ | % $K_2O$ | % $K_2O$ |
| A. River Thames, Abingdon | 1·73 | $<$0·0001 | — | — | — |
| B. Airfield soil, Harwell | 2·27 | 0·00057 | 0·0028 | 1·07 | 1·06 |
| C. Granite, Malvern | 3·18 | 0·0030 | 0·014 | 5·4 | 1·30 |
| D. Keuper Marl, Taunton | 4·00 | 0·0035 | 0·017 | 6·7 | 2·72 |
| E. Slate, Dolgelley | 4·09 | 0·0039 | 0·018 | 7·0 | 1·53 |

Background measurements were made by placing the probe unit on open flat ground in order to obtain ratemeter readings corresponding substantially to a solid angle of $2\pi$. Owing to the severe gamma-ray absorption in the ground, the ratemeter responded inappreciably to activity in soil further than about 30 cm. from the probe. Provided the soil was homogeneous and free from vegetation, a sample representative of the effective volume of soil could be removed for laboratory assay. The soil samples were assayed by a standard radiometric method (Franklin and Barnes 1948), the results being expressed in terms of *alternative* percentage concentration of $U_3O_8$, $ThO_2$ or $K_2O$, corrections being made in each case for self-absorption.

The ratemeter readings converted to a specific counting rate scale (counts per minute per cm$^2$ cross-sectional area) are given in the Table. Negligible activity was assumed for case A, where the probe was held over the surface of the River Thames from a small wooden boat. The assayed concentration figures have a standard deviation of $\pm 3\%$: this is presumably smaller than the sampling errors. The specific counting rate with the probe enclosed in a cylindrical 'aged' lead shield of 5 cm. wall thickness was 1·36, about three-quarters of 1·82, the rate extrapolated to zero soil activity. In the final column of the Table are listed the results of a spectrographic analysis, with an accuracy of within

15%, for $K_2O$. Thus the activity of sample B is practically wholly accounted for by $K_2O$, whereas in the other samples the balance of activity is presumably made by $U_3O_8$ or $ThO_2$ or both.

That contamination and atmospheric activity contribute negligibly to the residual ratemeter reading at zero soil activity may be indicated as follows. Careful inspection and analysis of the Geiger counter materials showed no appreciable activity due to contamination. The specific counting rate at ground level corresponding to a mean radioactivity in the lower atmosphere of $3 \times 10^{-13}$ curies per litre is calculated to be not greater than 0·03. It is reasonable to accept a value of $1{\cdot}80 \pm 0{\cdot}2$ (including the effect of barometric variations) as due to cosmic-ray activity measured at a few hundred feet above sea level and in latitudes around 52°. The specific gamma-ray background counting rate of a Geiger–Müller counter type CV2148 may be derived from the Table. Upon soil having an equivalent $U_3O_8$ concentration of 0·001% by weight this rate is $0{\cdot}6 \pm 0{\cdot}06$. This is superimposed upon the cosmic-ray contribution and refers to conditions of $2\pi$ solid angle. The value of the cosmic rate is consistent with a unidirectional intensity of 1·2 per $cm^2$ of horizontal plane area per minute (Montgomery 1950).

Acknowledgments are due to Mr. R. P. Thorne for the spectrographic determinations and to the Director, Atomic Energy Research Establishment, for permission to publish this letter.

Atomic Energy Research Establishment,
Harwell, Didcot, Berks.
18th February 1951.

D. H. Peirson.

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# REVIEWS OF BOOKS

*Practical Physics*, by Sis Cyril Ashford. Pp. viii+173. 1st Edition. (Cambridge : University Press, 1950.) 10*s*. 6*d*.

The author of this textbook is a teacher of wide experience, and this is revealed in the thought which has been given to the method of presentation. Sir Cyril has based his book, with the encouragement of the Local Examinations Syndicate, on the practical exercises set in the Cambridge Higher Certificate examination between 1932 and 1947. The suitability of the experiments as a laboratory course for students taking this examination cannot, therefore, be questioned, although the book's wider appeal may be restricted by the bias thereby imposed. For example, out of fifty experiments described, only twelve are electric or magnetic in character, whereas experiments on light occupy more than one third of the descriptive space.

As to be expected when the same experiment has to be performed in a wide variety of school laboratories, the apparatus described is very simple in character, and Sir Cyril makes a highly commendable point in stating that the student should rely on his own powers rather than on elaborate equipment. He supports his principle by quoting C. V. Boys' words not " to bow down to the brazen image that the instrument maker has set up ", and adds that the student should make the very best use of what is to hand, however simple it may be.

The section on lenses is unusually comprehensive for a book of this standard, and the description of the experimental apparatus and procedure is backed by the essential theoretical notes; there are further supporting appendices at the end of the book with additional theory and a most useful account of the manner in which spherical aberration affects the use of the parallax method in optical measurements.

Although the experiments are, in general, 'old friends', a number are presented in an unusual way to attract the student's attention to underlying principles; for example, the simple experiment normally used to illustrate the working of the hot-wire ammeter is described as a method for verifying Newton's law of cooling—though no forced draught is provided !

The book is prefaced by a useful introduction on the reduction of observations, in which the current fashion of graphical plotting to obtain linear relationships is emphasized. The only minor blemish noted in this carefully produced book is the omission of units on a few graphs, e.g. Figures 2, 49 (*a*), 49 (*b*), and in the tables on page 153. R. W. B. S.

### *Textbook of General Physics*, by G. R. NOAKES. Pp. viii + 415. 1st Edition. (London: Macmillan and Co., 1950.) 10*s*. 6*d*.

We are entitled to expect competence in any textbook of physics at the advanced and scholarship level, but it needs something more than competence to command the wholehearted welcome which we feel this volume merits. Within the essential framework common to all who cover this ground there is ample room for teaching skill and style by which an author may succeed handsomely and deserve applause. Let us say at once that this is such a book.

The author already has volumes on light, electricity and magnetism, and heat which are widely used. He now deals with mechanics, properties of matter and wave motion. Believing that students find the mathematics difficult because they do not grasp the physical facts, he establishes his structure by appeals to experimental facts rather than by appeals to intuition—which is as it should be. This approach is reinforced by such devices as introducing the student to *Principia Mathematica* in translation and with modernized terminology and notation. The result is an excellent discussion of the laws of motion, in which, as it were, the student meets Newton in the act of discovery.

There is a welcome section on the ways in which laws may be verified (and, later perhaps, discovered) by graphical methods. There will be some readers who will think that the determination of the constants in laws of the type $P=kQ^n$ is not dealt with in the best fashion for those students who are uncomfortable with the plotting of logarithms with negative characteristics. All such difficulties vanish if the variables, $P$ and $Q$, are coded by multiplication by powers of ten so as to eliminate the negative characteristics. Moreover, the procedure for fitting a straight line by least squares is arithmetically simple and might well now be taught, since simple statistics have been introduced into the Intermediate Science syllabus, presumably to give students better ability in handling experimental data. Outside the school laboratory results do not always lend themselves to fitting a line by eye.

Friction and stresses and strains are splendidly treated. It is refreshing to see a school text openly questioning whether the observed departure from Hooke's law under light loading conditions is simply due to initial kinks in the wire, and stimulating to see rubber, a very queer but essential substance, discussed on the basis of the modern work of Treloar and his colleagues. This and other quotations from current research bring home to the student that even 'elementary' physics is still a matter for investigation.

The author's comment on the significance of terms in an equation (always a good test of an expositor's skill) can be very apt: " . . . . if $a/V^2$ (in the van der Waals equation) is greater still, $p$ is zero and the container can be dispensed with, so the substance is a solid ". On the other hand, the author slips from grace, surely, when he says $p=2vmu^2/6$ is " nearer the truth " than $p=2vmu^2$ for the pressure of a gas on the wall of its container ?

There is a good section on wave motion which is rightly treated as a major topic, rather than as an incidental to sound and physical optics, and a substantial rounding off chapter on the physics of $G$ and $g$.

A comprehensive range of examples from Higher Schools and Scholarship examinations is fortified by answers and, where suitable, partial solutions. Logarithmic and trigonometric tables are given. The index stood up to a stiff test, but Poisseuille's formula will be found on page 249, not 240. The production is a credit to Messrs. Macmillan as well as to the author. It is confidently recommended. M. J. M.

### *Mechanics*, by M. SCOTT. Pp. xi + 394. (New York: McGraw-Hill Book Co., Inc., 1949.) 38*s*. 6*d*.

This book on mechanics by Merit Scott follows conventional lines and is divided into approximately two equal sections on statics and dynamics respectively. No mathematics higher than the calculus is employed, and the book should provide a sound basic course for a first year physics honours undergraduate.

The author includes both the long form notation and the vector notation, with simple vector operations which are explained in Chapters I and II. He lays great stress on the fact that the problems associated with each chapter are an essential part of the whole text, but one might have hoped that, coming from a physicist, the examples would have been less artificial and more of a physical nature. Moreover, the majority of numerical questions are expressed in lb.ft. units, but one is surprised to find that $g = 980{\cdot}8\,ft/sec^2$ (Problem 23.5, page 354).

It is stated in the preface that the book is written so as to enable a good student to use it unaided, and to fulfil this purpose more effectively the author might profitably consider the inclusion, in future editions, of answers to the problems. R. W. B. S.

*Calculating Instruments and Machines*, by D. R. HARTREE. Pp. ix + 138. (Illinois: University Press, 1949; Cambridge: University Press, 1950.) 21*s*.

This book arose out of a course of lectures on computing equipment given by the author in 1948 at the University of Illinois. In an introductory chapter Professor Hartree defines his two main types of equipment. He has discarded the terms 'analogue' and 'digital' machines, which were introduced in America, in favour of the older terms, calculating instruments and machines to which they are roughly equivalent. The remainder of the book falls into two main sections which are almost independent; the first deals with computing instruments in general, and the second with a particular section of the class of calculating machines, the automatic general purpose machines.

It is natural that the section on calculating instruments should be devoted mainly to the differential analyser since it is the only equipment in this class which has a wide range of application. In an interesting historical introduction the author shows that the fundamental ideas involved in the construction of a differential analyser were clearly appreciated by Lord Kelvin as early as 1876 though little was done towards their realization until more than fifty years later. The general structure and principles of a differential analyser are described in terms of the earlier mechanical models with particular reference to the machine installed at the University of Manchester with which the author was associated. This part of the book is particularly lucid and satisfying. An indication of the manner in which the mechanical equipment has been replaced by electrical counterparts in the newer differential analysers follows.

A second chapter on the differential analyser describes the techniques which have been developed, mainly by the author's group at Manchester University, for dealing with second order partial differential equations in two independent variables. Examples are given of the solution of parabolic partial differential equations by replacing the derivatives with respect to one of the independent variables by finite differences, and a brief comparison is made of the relative merits of making this substitution for each of the two independent variables. The use of the differential analyser for solving hyperbolic partial differential equations by the method of characteristics is described with reference to two particular problems in compressible flow, one of axially symmetric isentropic flow around a solid of revolution and the other for non-steady one dimensional flow.

The third chapter on calculating instruments gives a brief description of a number of special purpose machines including instruments for solving sets of linear equations, locating complex roots of polynomials, evaluating definite integrals and performing Fourier syntheses.

Professor Hartree introduces the section on general purpose computers with a note on terminology which should be very helpful to those to whom the subject is new. Before dealing with any specific machines he gives a fairly detailed discussion of the main features which are common to all general purpose machines. The next three chapters describe, in chronological order, the various attempts which have been made, and which are being made, to build machines to realize these objectives, from Babbage's Analytical Engine to the modern electronic high speed computers.

The chapter on Babbage's Analytical Engine should prove a useful corrective to the tendency to regard most of the basic ideas involved in the design of general purpose machines and, more particularly, in coding and programming for them, as being of recent origin. The author makes it quite clear that Babbage gave a considerable amount of thought to all these aspects of the problem, and that it is the exploitation of the recent developments in the fields of electrical and electronic engineering which chiefly accounts for the superiority of modern machines over the Analytic Engine.

The following chapter is devoted largely to a description of the Harvard Mark I machine, the first machine to realize Babbage's main objectives, and of the Eniac, the first electronic computer. The capabilities of the Eniac are illustrated by means of a problem in the theory of the laminar boundary layer in a compressible fluid which was solved by the author on this machine.

A third chapter on general purpose machines covers most of the main projects in the U.S.A. and this country at the present moment. These projects are too numerous for a detailed description to be given of each one, but a fairly clear picture of their main points of similarity and the range of variation is achieved. A section of this chapter is devoted to the use of the diagrammatic notation, introduced by von Neumann and extended by Turing, for the functional analysis of serial machines, but here the treatment suffers somewhat from overcondensation.

The book concludes with a brief review of some aspects of numerical analysis with special reference to the re-orientation which the new high speed machines will make necessary.

The author's treatment of his subject throughout is such that it will appeal to mathematical physicists rather than to those whose primary interest is in constructing equipment. The book should prove invaluable to the graduate who is interested in numerical analysis and wishes to gain a clear idea of the main trends of modern developments in this field.

J. H. W.

*Shorter Six-figure Mathematical Tables*, by L. J. COMRIE. Pp. xxvi + 387. (Edinburgh and London: Chambers Ltd., 1950.) 12*s*. 6*d*.

If we divide tables of functions into two classes, the general ones and the special ones, then logarithms, trigonometrical functions, squares and square roots will certainly fall into the first class, and $Hh_n$ functions for high $n$ into the second. We do not want books in which a number of different special tables are collected together, but experience has shown that collections of general tables are very useful, and it is of these that we shall be speaking in the remainder of this review.

As Dr. Comrie remarks, four-figure tables are used much more widely than any others, at any rate for most scientific and engineering work, though in navigation it has been usual to use five figures and in astronomy to use seven. A very convenient set of seven-figure logarithms was issued by Messrs. Chambers in 1844 and this collection, in the form in which it was issued in 1878, became by far the most popular volume for use when four figures did not suffice. Its arrangement of trigonometrical functions was not particularly good, and it gave few of the functions, such as the hyperbolic functions, which are now in frequent use, preferring to use the space for a table of quarter-squares (which can be used for multiplication) and for traverse tables (tables of $a \cos \theta$ and $a \sin \theta$ for different values of $a$ and of $\theta$).

Not long ago, Dr. Comrie prepared for Messrs. Chambers a new set of six-figure tables to replace this old volume, and also a 64-page set of four-figure tables, which in a sense represented a condensation of the larger work. The volume now under review is more truly a condensation. Like the full work, it consists of six-figure tables, but with certain omissions and, in places, with increase of the tabular interval. He has now given us logarithms, natural logarithms, trigonometrical functions of angles, expressed either in radians or in degrees and minutes, the logarithms of these functions (but only for angles expressed in degrees and minutes), hyperbolic functions and powers and roots of integers. For the lower integers, these go up to the fifth power and root, and include the negative powers $-\frac{1}{2}$, $-1$ and $-2$. For the higher numbers, they stop at the fourth power and the cube root and the simple reciprocal is the only negative power. For all integers up to 1,000, the factorial and the prime factors are given, and there is a list of primes up to 12,919. There are no gamma functions.

Great care has been taken to assist the reader. The type chosen is a very legible old style, and the use of rules has been kept to a minimum, with a consequent reduction of strain on the reader's eye. First differences are given in many tables, and they are printed in italic if second differences are not negligible. Proportional parts are printed as required on the pages occupied by the logarithm table, and a complete $3 \times 1$ table of proportional parts occupies pages 358 to 377. There are also tables of interpolation coefficients in Bessel's formula.

The whole volume is very attractive, and, at its price, which corresponds to about 5*s.* before the War, should become very widely known and used. J. H. A.

### *Journal of Atmospheric and Terrestrial Physics.* Volume 1, No. 1, 1950. Edited by Sir EDWARD APPLETON. (London: Butterworth-Springer Ltd.) 70*s.* per volume.

The *Journal of Atmospheric and Terrestrial Physics*, of which the first issue has now appeared, is under the Editorship-in-Chief of Sir Edward Appleton, who is assisted by an editorial advisory board, the composition of which is clear evidence of the favour with which this new publication is regarded by leading workers in the various fields concerned in many countries.

Recent experience of journals restricted in scope and with international support has been decidedly favourable, and although the foreseeable price of success—that all the best papers in a certain field may be withdrawn from journals which workers in other specialities read—is to be deplored, this probably represents the most promising line of rationalization of publication at the present time. It is of course true that the multiplication of these specialized publications in some degree brings into question the place of the established journals of the learned societies.

In the present instance, there can be no doubt that the scope of the new journal covers material previously found in several, sometimes even inappropriate, periodicals of a single country alone, and the conveniences of concentration now to be expected are likely to be marked.

The scope of the first number is sufficiently indicated by the contents list, which includes papers on the 27-day variation of the F2 layer (Bartels), the ionization balance in the atmosphere (Hess) daily variations of atmospheric conductivity (Israël), tropospheric reception (Pickard and Stetson), the chemical analysis of stratosphere samples (Paneth) and periodic solar cosmic-ray relationships (Hogg), in addition to a most interesting collection of observations reported by W. Dieminger and his co-workers on phenomena associated with the solar disturbances of 19th November 1949.

The publishers have achieved an attractive and individual format for the *Journal*, including among other desirable features the uniform drafting of line diagrams. To what extent the avowed aim of speedy publication will, in fact, be maintained remains to be seen, but if this can be accomplished, the future of the *Journal of Atmospheric and Terrestrial Physics* should be assured. J. G. WILSON.

### *The Flight of Thunderbolts*, by B. F. J. SCHONLAND. Pp. 152. (Oxford: Clarendon Press, 1950.) 15*s*.

This book is correctly described on the dust cover as providing " for the general reader a scientific ' case history ' of much interest, and for the meteorologist, physicist, and electrical engineer, the first full historical and general account of the subject in English ". In these days most scientific books are suitable only for those already interested in the particular subject : too few are written which excite the interest of the general scientific reader or specialist in other branches, and which at the same time introduce him to those problems still requiring research. This is such a book. It covers approximately the ground of the author's Charles Chree Address to the Physical Society.

Lightning is of obvious practical importance, not least to the power-distribution engineer, but, in addition, it poses problems in physics still unsolved, one of which is the precise mechanism of the pre-breakdown ionization currents which lead to the lightning flash. In view of its enormous destructive power, it is hardly surprising that lightning has always been a source of fear to man, and that its action has long been associated with the wrath of Gods. This ' Thunder-Magic ' of primitive man is traced, in the first chapter, through Greek and Roman times, to the eighteenth century, when scientific experiments with lightning began. This chapter also contains interesting early records of damage by lightning to famous buildings.

The second chapter is devoted to the invention of the lightning rod by Franklin, and the third chapter deals with the various forms taken by the flash: it is surprising to be told that no authenticated case of so-called ball-lightning exists. It is also pointed out that it "is possible that the burning yet unconsumed bush encountered by Moses on Mt. Sinai was St. Elmo's fire" (corona glow discharge).

The remaining five chapters deal with the luminous and electrical processes in the flash, lightning protection, thunderclouds and the effects of thunderstorms. Most of the experimental investigations have been carried out by photographic means, following the pioneer work of Kayser; the Boys camera, together with its later developments, and the analysis of the photographs obtained, are fully described. It is, perhaps, a pity that, for the sake of completeness, the chapter on the discharge processes did not deal more adequately with the nature of the important problem of the pre-breakdown conductivity, but, as the author himself says in the preface, "the book is not a detailed monograph, and the more complicated aspects of the problem have been touched upon rather lightly". Nevertheless, the book, written by one who has himself done distinguished work in this field, makes altogether delightful reading.

F. LLEWELLYN JONES.

*Progress in Chromatography* 1938–1947, by L. ZECHMEISTER. Pp. xviii+368. (London: Chapman and Hall 1950.) 45*s*.

This work serves as a supplement to Zechmeister and Cholnoky's *Principles and Practice of Chromatography*. The ten years which it covers have seen considerable advances in the field. Though in note form, and consequently not very readable, the second and main part of the book, dealing with the different groups of chemical substances, provides a wealth of information not easily available elsewhere. The first part, dealing in a general way with principles and methods seems to the reviewer less happily devised. There is a widespread tendency to regard chromatography in a mystical spirit and to say that its theory still remains to be elaborated. In fact, it is deficiency in the techniques of mathematics that prevents calculation of how substances will behave on chromatograms—the physical processes that must be taken into account in setting up the fundamental equations are fairly well understood. Consequently the mathematical treatments have so far dealt only with certain special cases, admittedly of practical importance. Arguments (concealed under a cloak of algebraical symbols) as to which physical factors can be ignored in order to simplify the mathematical treatment have been misconstrued as arguments about the fundamental nature of chromatography. Professor Zechmeister has not clearly distinguished these aspects of 'theory', or he would not have written (page 4) "... theoretical efforts made so far have not influenced experimental work". The theoretical principles of continuing interest to chromatographers are those determining the distribution of substances between the different phases of the chromatogram, and their dependence on molecular structure. Professor Zechmeister deals shortly with only a few aspects of this subject—such as specificity in adsorption of *cis-trans* isomers. Of course, any general treatment of intermolecular forces would have extended unreasonably the scope of the book, but the topics selected for discussion in the first part of the book perhaps give too limited an idea of the scope of chromatography. However, any such impression is dispelled by the survey of chromatographic achievements set out in the second part. As a work of reference this book will be consulted for many years to come for its comprehensive and orderly treatment of the literature of the period. The book is appropriately dedicated to the memory of M. Tsvet, the discoverer of chromatography, whose photograph serves as frontispiece.

R. L. M. SYNGE.

*Physico-Chemical Constants of Pure Organic Compounds*, by J. TIMMERMANS. Pp. viii+679. 1st Edition. (Amsterdam: Elsevier; London distributors: Cleaver-Hume Press, 1950.) 95*s*.

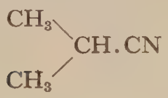
"This work records, as completely as possible, the physico-chemical constants of organic compounds which have been measured with sufficient care to warrant their acceptance as data established with a precision worthy of contemporary science." These are the opening words of the introduction to this valuable book of reference.

The constants recorded, when reliable data exist, include critical constants, densities of liquid and saturated vapour, saturated vapour pressures, boiling-point, freezing-point, triple-point, density, viscosity, surface tension, refractive index, dielectric constant, specific heat, heat of vaporization, heat of melting, heat of combustion.

This book has one admirable feature which places it on a higher plane than any other reference book on similar properties. A certain standard of reliability and accuracy is laid down by the author. All available measurements judged as coming up to this standard are included in detail with full references; all measurements judged as not coming up to this standard are ignored. All serious chemists are under an immense debt of gratitude to Professor Timmermans for having accomplished this prodigious task of separating the grain from the chaff, even if it should later be discovered that a small trace of chaff has inevitably been left with the grain.

By contrast I have, unfortunately, not a good word to say for the editing, which is largely conspicuous by its absence. The most serious crime is that each table is not labelled with the name of the substance to which it refers. Thus pages 140–153 are devoted to benzene and to toluene, the two sets of tables being separated by the single word TOLUENE on page 150. The danger of accidentally using a table relating to the wrong substance is tremendous. This is by far the worst, but not the only, evidence that the editor is either surprisingly irresponsible or morbidly parsimonious. When there is not room in a table for a long word it is usual and justifiable to use an abbreviation, but the use of 'recalc.', 'corr.', 'unp.' and 'interp.' when there is no lack of space is 'comp. inexc. & indef.'. Nor is it fair to expect the user of the book to know what is meant by the remark "the fifth degree of purity can be easily attained". Nor is there any justification for using $c_p$ as an abbreviation for 'at constant pressure'. Another example of exaggerated condensation amounting to inaccuracy is on page 24, where the recipe given for obtaining pure *ethane* is by the action of metallic sodium on *acetonitrile*!

I found the inconsistency (and sometimes obscurity) in the setting-up of formulae extremely irritating. Here are a few examples :



$(CH_3)_2CH.CH_2CN$ $RCH(CH_3)CN$

$CN.C(CH_3){=}CH.CN$ $CN.CH_2.CN$ $N{\equiv}C.OH$ $CNCl$

$C_6H_5NH.CH_3$ $C_6H_5.N(C_2H_5)_2$ $O{-}C_6H_4$ $CH_3.NH_2$

$C_4H_3(CH_3CO)S$ $C_6H_8(CH_3)_2$

$C_5H_4N.C_4H_7N(CH_3)$

If there is any systematic principle, it would seem to be that the more complicated the formula the less one should bother to facilitate its recognition.

The text occupies only a minute part of the book. The translator's style is sometimes quaint. For example, on page 19: "The hydrocarbons . . . are a first rate physico-chemical material", and on page 633: ". . . have studied the variation of entropy . . .; in particular Aston . . . and Giauque were very *fortunate* in this field.".

I have made no systematic search for misprints, but in a casual glance through the book I noticed four :—

| | |
|---|---|
| p. 28, 2nd line : | Corrects should be corrects; |
| p. 139, 3rd line from bottom : | 130° and 110° should be −130° and −110°; |
| p. 215, 2nd table : | $d_p$ should be *dp*; |
| p. 245, top : | $(CHBr_2)$ should be $(CHBr_2)_2$. |

Misprints of this type are admittedly trivial, but they do suggest that the proof-reading was not of quite the highest standard required in a book of reference.

I can only express the hope that future editions will be edited in a manner worthy of the author's effort in collecting together such a mass of valuable information. E. A. G.

*Some Recent Developments in Rheology.* (Based on the 1949 Conference of the British Rheologists' Club.) Edited by V. G. W. Harrison. Pp. vii + 140. (London: United Trade Press Ltd., 1950.) 20*s.*

The organizers of Annual Conferences of the smaller scientific societies, especially those which, like the British Rheologists' Club, cater for a very wide range of interests, are faced with a situation not unlike that of Heissenberg's Uncertainty Principle. The more closely the subject-matter of the Conference is unified into a coherent whole, the greater the proportion of members who will have no direct interest in the subject: the greater the attempt to interest all the members, the less will be the chance of producing a concrete and convincing set of conclusions from the meetings. Moreover, the 'quantum' is relatively large, since, in the short time available, not more than ten or a dozen papers can be adequately read and discussed.

The problem of publication is also a difficult one. Small societies cannot afford to publish their own Proceedings and commercial publishers are loath to give prompt attention or high priority to books whose appeal must, in the nature of things, be somewhat restricted.

The previous publications of the Proceedings of the Annual Conferences of the British Rheologists' Club have had a somewhat chequered history. In some cases there have been such delays as seriously to prejudice the value of the work; but the publishers of the Proceedings of the 1949 Conference held in Bristol are to be congratulated on having completed publication before the opening of the 1950 Conference. On the other hand, the space available was so restricted that the discussions on the papers, which constitute perhaps the most valuable feature of such gatherings, could not be included and some of the papers themselves have had to be drastically curtailed.

In 1949, it was decided to include a very wide range of subject matter in the programme, as is indicated by the general title. The book should thus be regarded as a series of separate essays rather than as a unified whole, though, in a few cases, pairs of papers may be taken together; for example, F. R. N. Nabarro's and C. Crussard's papers on grain boundaries in metals and, rather more loosely connected, R. Harper's paper on psycho-physical and other psychological applications and G. W. Scott Blair's on some denotative and connotative aspects of rheology. Crussard's paper is published in French, thus avoiding the necessity of solving many vexed problems of technical translation.

W. W. Barkas' contribution on fibre assemblages presents an admirable illustration of the elegant application of classical theory to industrial problems and links up well with H. F. Rance's less theoretical treatment of the rheology of paper. The editor has had the thankless task of condensing the latter paper, since the author was ill at the time and was therefore unable to undertake this work for himself. Dr. Harrison is to be congratulated on the way he has done this, as on the rest of his work as editor.

H. A. Nancarrow and E. B. Atkinson contribute a useful practical study of the extrusion of thermo-plastics, linking with a rather similar paper by J. S. Gourlay and P. S. Williams, on concentrated dispersions of solid particles, mainly paints.

R. M. Davies' paper on internal friction of solids emphasizes the importance of rheology to engineers which, as a speaker at the 1950 Conference pointed out, is not yet adequately appreciated.

The last chapter of the book consists of an account (edited by Dr. J. G. Oldroyd) of an informal discussion on large elastic strains. This subject is of direct interest only to a limited number of members, since many rheologists lack the mathematical equipment needed to follow the controversial issues discussed; but the questions are of fundamental importance and their solution would benefit many who may have to take some of the arguments on trust.

The book as a whole forms a worth-while contribution to rheology but it is to be hoped that the Club (or the British Society of Rheology as it is in future to be called) will find some way of establishing a continuity in the publication of its Proceedings.

Rheologists will welcome the present rather heterogeneous collection of volumes on their shelves but it would be much appreciated if the Proceedings could appear promptly each year in uniform style and under the impression of a single publisher. G. W. S. B.

*Progress in Coal Science.* Edited by the late D. H. Bangham. Pp. xi + 456. (London: Butterworth's Scientific Publications, 1950.) 40*s*.

Coal is a sedimentary rock, formed from the remains of plants ; hence coal falls within the province of the geologist, the petrologist and the palaeobotanist. Coal is the most important fuel available to man at the present day, a fact which brings it within the sphere of the chemist, the physicist and the engineer. Coal may also be regarded as a raw material of the chemical industry ; therefore the chemist has a further claim. Moreover, certain aspects of the utilization of coal bring it within the sphere of the physiologist, the agriculturalist and the horticulturalist, for misuse of coal is a potent cause of sunlight starvation, particularly in the more highly developed areas of the earth's surface. If ever, therefore, a 'science' of coal is established, this will only be accomplished by the joint efforts of workers in a number of pure and applied sciences.

The past decade has seen a great quickening of interest in various aspects of coal, both in its scientific and technological relations ; not only has a greater volume of research been carried out, but the basis of that research has steadily widened. New techniques have been devised, and existing techniques have, for the first time been applied to problems arising in connection with coal. Most of the work done from 1940 onwards could not be published at the time, partly for reasons of security, partly for lack of time for the preparation of the necessary memoirs, and partly from scarcity of the materials required to print those memoirs. From 1943 to 1947, a number of reports of progress in various fields of coal science appeared in the *Monthly Bulletin of the British Coal Utilisation Research Association*, a publication which has but a limited circulation. These articles, together with one or two published elsewhere, form the greater part of the volume under review, the object being to make the articles available to a wider circle of readers than the rather favoured group who receive the *B.C.U.R.A. Monthly Bulletin*. The book is therefore a progress report, giving an account of our knowledge of certain aspects of coal and allied subjects at approximately the dates of the various articles, namely 1947 or 1948 ; it is not a textbook, for each chapter stands complete in itself, and there is no attempt at synthesizing the matter into a coherent story.

The book is divided into five parts, each having a separate title-page with a blank back, so that 10 of the 456 pages which comprise the text are thus employed. Part I is devoted to modern experimental techniques which are proving or which may prove valuable in the solution of problems arising in connection with fuels generally. The methods outlined include, x-ray crystallography, absorption spectroscopy, mass spectrography, chromatography, photographic studies in combustion, and measurement of the emissivity of flames.

The increasing importance of pulverized coal is the reason for Part II, which is devoted to discussions of the importance of fine powders and particles in industry, the measurement of their size and specific surfaces, to non-mechanical methods of size reduction and to the assessment of the 'grindability' of coal.

The constituents of coal are treated in the four chapters of Part III, which cover the formation of humus and its relation to coal, the analysis and sampling of solid fuel (a curious reversal of the natural order of procedure), the occurrence of rare elements in coal ashes, and the petrology of coal.

Part IV, on the organic chemistry of coal products, gives a useful summary of work on the action of chemical reagents on bituminous coal, and on the oxidation of coal at moderate temperatures. The progress of research on coal-tar from 1910 to 1940, and more recently is outlined in two chapters, while the remaining chapter of the section gives an outline of work on the synthesis of hydrocarbons.

The final section of the book consists of six chapters on chemical aspects of combustion and gasification, and it is here that the contrast between the 'classical' conceptions of the 1930's are most strongly contrasted with the present ideas. There is welcome evidence that the importance of the inorganic components of coal is recognized, and that the mineral matter can be deleterious not merely as a diluent and possible producer of clinker.

The book is a welcome addition to the literature of one important branch of Fuel Technology, and it is to be hoped that some sort of similar report may be issued (say) quinquennially. The broadening of the bases on which the investigations of coal and its products are founded is evidence of the increasing appreciation of scientists in all branches of the importance of our most valuable raw material. *Progress in Coal Science* is a necessity in the library of everyone who professes more than the most elementary interest in coal. One very valuable feature is the copious lists of references which, although admittedly not complete, are sufficient to provide the enquirer with occupation for his spare time for a long period.

G. W. HIMUS.

### *Progress in Biophysics and Biophysical Chemistry*—I, edited by J. A. V. BUTLER and J. T. RANDALL. Pp. viii + 279. (London: Butterworth–Springer, Ltd., 1950.) 50*s*.

The book is written primarily for the specialist and its appeal and value to him is obvious. It contains nine up-to-date reviews of selected and representative topics in biophysics written by prominent workers in the field. All reports are provided with probably complete references at least up to 1948 and some up to 1950 and they will certainly assist every biophysicist and biochemist in surveying his own research in the light of his colleagues' efforts.

To the non-specialist, such as the reviewer, the book presents a fascinating variety show, seen on a holiday, in a language only incompletely mastered. Naturally, at the time when he sees a number of his most brilliant contemporaries turning to biophysics he wants to know what it is. This book will be an admirable guide, because no attempts are made to make it easy by glossing over real difficulties in the discussions.

One of the topics of biophysics is the development and technology of instruments used in biological studies. There is a very brief review of the phase contrast microscope by A. F. W. Hughes, a fundamental, almost esoteric, study of the visibility of transparent media, *Local Refractometry: The Observations of Transparent Inhomogeneous Matter*, by J. St. L. Philpot and a detailed and very illuminating report on micro-radiography, *The Use of Soft X-Rays in the Assay of Biological Materials*, by A. Engström.

Secondly, there are three papers on what a short while ago were physiological problems: on the muscle as power plant by M. G. N. Pryor entitled *Mechanical Properties of Fibres and Muscles*, on the living body as a generator of electricity by E. E. Crane entitled *Bioelectric Potentials, their Maintenance and Function* and a paper by J. F. Loutit on *The Tolerance of Man for Radioactive Isotopes*. The first two articles in particular make clear to the physicist a particular origin of biophysics : a branch of physiology that has become too difficult for the physiologist and a branch of physics that has become too complicated for the physicist ; in fact a great deal of the context will elude many physicists simply by their inability to follow the terminology. In spite of this difficulty they will realize how much progress has been made in the understanding of the animal power and electricity plant and how little is known.

A third aspect of biophysics is offered by the fact that life is based on molecules such as polypeptides, polynucleotides, polysaccharides and proteins whose very size and molecular weight preclude their investigation by ordinary chemical methods and possibly by any method. What can be done and has been achieved mainly by X-ray analysis and by the study of the properties of solutions is discussed in three articles ; *Fundamental Structures in Biological Systems* by K. M. Rudall, *Properties of Solutions of Large Molecules* by H. Gutfreund and *Scattering of Visible Light and X-Rays by Solutions of Proteins* by G. Oster. Again one wonders whether more to admire the knowledge gained in spite of adversity or the courage of the investigators while the aim is still so remote.

The swing to biophysics now resembles the swing to nuclear physics 25 years ago. Will it bear similar fruit ? One is looking forward to future volumes of the series for the answer.

W. E.

### *Analytical Absorption Spectroscopy*, edited by M. G. MELLON. Pp. vii + 618. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1950.) 72*s*.

Most scientists probably regard absorption spectroscopy as a highly specialized subject, yet such are the super-specialist times in which we live, that it has been found necessary to call on no less than nine contributors to compile this volume. It is, indeed, a comprehensive account of the subject, as can best be illustrated by giving the chapter headings and authors :

1. Chemistry : Preparation of Systems for Absorptimetric Measurement—M. L. Moss. 2. Physics : General Principles of Absorptimetric Measurements—M. G. Mellon. 3. Color Comparimeters—W. B. Fortune. 4. Filter Photometers—R. H. Müller. 5. Spectrophotometers : Ultra-violet and Visible Regions—K. S. Gibson. 6. Photographic Methods—E. R. Holiday. 7. Applications of Ultra-violet and Visual Spectrophotometric Data—E. I. Stearns. 8. Spectrophotometers : Infra-red Region—L. J. Brady. 9. Measurement and Specification of Color—D. B. Judd. All the contributors derive their authority from intimate practical experience of the matters on which they write, and it is gratifying to find that this country is represented by Dr. Holiday's contribution.

Perhaps the most valuable feature of this book is its critical and unprejudiced assessment of the relative merits of different spectrophotometers and spectrophotometric methods for the various types of problem that have to be solved. The wavelength range dealt with is from $0{\cdot}2\mu$ to $25\mu$, and instruments suitable for one part of this range may not be suitable for other parts ; different optical materials, for example, and different radiation detectors may have to be used. On the other hand, in some regions two or three alternatives may be open to the investigator, and it is very helpful to have a detailed comparison of the precision and accuracy of measurement that may be attained, and also information about the practical convenience of each instrument, its speed of operation, flexibility, spectral transmission, freedom from stray light, dispersion, and so on.

Where so much is good, it would be invidious to pay special tribute to one chapter or section in comparison with others, and detailed criticism, too, is perhaps out of place. In any case, there seems to be remarkably little to criticize and hardly any of those small slips that reviewers delight to discover. Attention might, perhaps, be drawn to a misconception (p. 119) of the significance of the luminosity curve, since it is used to deduce those wavelengths for which the eye has good intensity discrimination. The conclusions may well be correct, but the premises are wrong.

The book is very well produced, has many excellent diagrams and each chapter has an extensive list of references. The price is, of course, the snag, but that is one of the penalties that has to be paid for devaluation. W. D. WRIGHT.

*Kleinste Drucke, ihre Messung und Erzeugung*, by R. Jaeckel. Pp. x+302. (Berlin : Springer-Verlag, 1950.) No price.

The use of high vacuum methods is no longer confined to physics laboratories, but has spread to many branches of pure and applied science and to engineering. The recent advances in high vacuum technique have mainly consisted in developments and refinements of existing methods and their adaptation to work on a large scale. The book by R. Jaeckel aims at giving a comprehensive picture of these methods.

The first chapter contains a brief outline of the relevant concepts and results of kinetic theory. The following chapter deals with the measurement of low pressures, and all the more important gauges are treated very satisfactorily. The subject of leak-testing has, unfortunately, not received the space and care which it deserves. Mass-spectroscopic leak-testing is described, but the statement that either $H_2$ or He can be used misses the point of this method, which allows the detection and measurement of the helium gas leaking into the apparatus, undisturbed by gases such as hydrogen which are always present unless the apparatus has been thoroughly outgassed. The simple and effective method of leak-testing by means of a Pirani gauge in conjunction with hydrogen as a test gas is not mentioned.

The chapter on vacuum pumps represents probably the most valuable part of the book. The theory of the vapour jet is given in some detail, and the much discussed mode of action of diffusion pumps is treated in a most thorough and lucid manner and illustrated by means of graphs and examples.

The following chapter describes such matters as outgassing, adsorption and gettering effects, vapour traps and clean-up in discharges, while the last two chapters deal with vacuum connections, taps, valves, greases and waxes. These chapters are not as up-to-date as the rest of the book. The subjects of waxes and metal–glass seals deserve a little more space; copper–glass connections (Housekeeper seals) are not even mentioned.

Such omissions and mistakes are, however, of minor importance compared with the merits of the book. It is written clearly and critically, and every new subject is introduced by an exposition of the underlying facts and ideas. Many graphs, such as those giving the pumping speed as function of tube diameter and pressure (pp. 293 and 294) will be greatly appreciated by the reader. This book will be welcomed by many physicists and engineers, especially those concerned with the design of vacuum equipment on a large scale.

H. KUHN.

### *Elements of Heat Transfer and Insulation*, by M. Jakob and G. A. Hawkins. 2nd Edition. Pp. xvi+230. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1950.) 32*s*.

Max Jakob is one of the foremost investigators in the field of applied heat, and is at present a professor of mechanical engineering ; his collaborator is Research Professor of Heat Transfer at Purdue. It is not surprising therefore that their book is written mainly for the engineering student, for whom the units generally used in it—British Thermal Units, degrees Fahrenheit and feet — will be at least as familiar as C.G.S. or M.K.S. units.

The matter dealt with can be described as the three modes of transfer of heat. The first of the chapters on conduction describes rather briefly the factors affecting thermal conductivity, but is very valuable for its compilation of data on insulators. It is followed by chapters on the calculation of conduction, either by simple analytical methods or by making use of experimentally determined 'shape factors', or by numerical methods such as the relaxation method or Schmid's graphical method. Methods of calculation are shown both for steady and for variable regimes, including cases where heat is generated in the body of the conducting medium, but no advanced mathematics is used, even the simple Fourier series being excluded.

Convection is treated, as it must be, by dimensional analysis, and selected equations are given for the commonest cases involving either forced or natural convection. The Reynolds analogy is referred to, but not treated in much detail, and the same is true of the film or boundary-layer concept. The chapter on the industrially important, and theoretically difficult, subject of heat transfer in condensing and boiling occupies only six pages, but manages to include useful data.

In the chapter on radiation, information not easily obtainable elsewhere is given on the effect of multiple reflection during exchange by radiation.

After a five-page chapter on the experimental determination of conductivities of metals and insulators, there is a short one on the errors which conduction can introduce in the measurement of temperature if proper precautions are not taken, and a final one on the Reynolds analogy.

The book should be valuable to engineering, and to some other, students, but it hardly gives them all they will need on the subject. Another blemish, which it shares with most modern American books, is a lack of interest in work not published in that country or in the country of origin of the author. Thus the work of Griffiths on insulators is not mentioned, nor is it suggested that the National Physical Laboratory was a pioneer of the methods described for measuring conductivity. Powell's work on the thermal conductivity of metals is not mentioned, though it is probably more accurate than the work cited. Even the name of Stanton, a pupil of Osborne Reynolds, who did so much to investigate his analogy, does not appear in the index. To counterbalance this blemish there is an admirable feature in the copious supply of examples for the student to work. J. H. A.

### *Heat and Temperature Measurement*, by R. L. Weber. Pp. vii+422. (New York: Prentice-Hall, Inc., 1950.) $6.65.

The student is now being well catered for as regards textbooks on Heat and Temperature Measurement and the volume under review is a useful addition to the series. A brief survey of the contents will give an idea of range of topics dealt with and as the author invites suggestions the opportunity is taken to indicate where additions and amendments are deemed desirable.

On the inside cover of the book is illustrated a number of 17th Century thermometers reproduced from a book by Fritz Burckhardt published in 1867.

The first chapter is devoted to the evolution of the concept of heat and temperature. The author has not yet adopted the term Celsius instead of Centigrade. The second chapter describes various forms of expansion thermometers. Here mention might have been made of a type occasionally seen in the holds of refrigerated ships. In this thermometer up to six bulbs each about 20 inches long and 1 inch in diameter are connected to a common capillary.

Chapter 3 is devoted to heat transfer. There one finds a description of the guard ring apparatus for thermal conductivity measurement, the theory of conduction in spherical shells and cylinders and periodic flow. It would be worth while adding a description of the apparatus used for the determination of the thermal conductivity of metal rods where a coaxial guard tube is used to eliminate correction for side loss. It is difficult to understand why the student should be burdened with the theory and technique of the Forbes' bar method, unless it is for examination purposes.

In chapter 4 thermoelectricity is explained whilst the following chapter is devoted to the thermoelectric method of measuring temperature. The theoretical laws of radiation are expounded in chapter 6 as an introduction to optical and radiation pyrometry in chapter 7. The modern form of disappearing filament optical pyrometer which has a potentiometer for the measurement of the current through the lamp filament is described.

Chapter 8 covers resistance thermometry, while in chapter 9 temperature recording and controlling are dealt with; the recorders described are, of course, of American type. An interesting feature of this chapter is the discussion of controller action: a subject not usually discussed in textbooks on heat.

The main theme of chapter 10 is calorimetry. There one finds descriptions of water, ice and aneroid calorimeters, but no description of electrical methods except Callendar and Barnes' continuous-flow calorimeter. To enable the student to understand precision calorimetric temperature measurement a description is given of the Mueller Bridge.

Chapter 11 covers phase diagrams and thermal analysis : it is a most helpful chapter to the student concerned with physical metallurgy.

Thermodynamics and allied topics such as the absolute scale of temperature are discussed in chapter 12, and in chapter 13 the production of extreme temperatures high and low is discussed. Here one finds descriptions of apparatus for the liquefaction of gases and the Collins helium cryostat. The magnetic approach to absolute zero is also discussed. Chapter 14 is devoted to special methods of temperature measurement such as flame temperature measurement, interferometric method, thermocolours, etc.

The concluding chapter of Part I explains the present position as regards the International Temperature Scale. A comparison of the scale of 1927 with that adopted in 1948 shows little change below the freezing point of palladium which value is reduced from 1,555 to 1,552 and that of tungsten is reduced by 20°c. The International Scale and the Thermodynamic Scale agree to better than 0·05° c. from 0° c. to the oxygen point but below this temperature the difference is appreciable.

Part II of the book cites a series of laboratory experiments; one of the experiments is the determination of the thermal conductivity of a gas by a comparison method. It is not clear how convection loss is allowed for in the technique described. Specific heats of liquids are determined by comparing the rate of cooling of the liquid with the rate of cooling of water. More precise methods might be described. In dealing with relative humidity the Assmann type of wet and dry bulb should be mentioned on account of its convenience.

Numerous tables are included at the end of the book. Some of these require revision. For example in Table 4, Properties of Materials, the boiling point of aluminium is given as 800° c. when the true value is about 2,060° c.; the density of aluminium is quoted as 8·40 instead of 2·7; brass is ascribed a thermal conductivity of 0·023 when the value is nearer 0·23 and the heat of fusion of iron is also in error by about 10-fold; the atomic weight of copper is given as 162·46 whilst in Table 3 it is quoted as 63·57; the atomic weight of molybdenum is given as 99·5 instead of 95·9, in Table 3 it is quoted as 96·0. The author should consult the *Metal Handbook* published in 1948 by the American Society of Metals.

The appearance of such errors in a textbook raises the question whether the author would be well advised not to use the space for numerous tables but refer the student to sources of information. Nowadays most students have access to tables of chemical and physical constants.

Finally in discussing the calorie the author should point out that by international agreement heat quantities should, whenever possible, be expressed in joules. EZER GRIFFITHS.

*Ionization Chambers and Counters*, by D. H. Wilkinson. Pp. ix + 266. Cambridge Monographs on Physics. 1st Edition. (Cambridge: University Press, 1950.) 25*s*.

The subject of electrical counting is very extensive if it is taken to include the many applications and associated circuits. It is necessary therefore that the scope of a monograph should be severely limited if more than a sketchy account is to be presented. The author has chosen to deal with the principles of operation of chambers and counters and the relevant theory ; particular problems are only mentioned as illustrative examples and no electronics is included.

Among the subjects treated in the introductory chapters are : energy range relation, straggling, ion and electron mobilities, electron capture and recombination. An excellent chapter is devoted to the electrostatics of pulse formation; it includes a discussion of the shape of the pulses from the various detectors, the effect of differentiation and the use of 'gridded' and thin wire chambers. Ionization chambers, proportional counters, and Geiger counters are dealt with in separate chapters. Each chapter is comprehensive, and the factors limiting the performance under various conditions are assessed. The book concludes with a useful chapter on the speed and statistics of counting.

In view of the title of this book, and the aim of this series of monographs, it is to be regretted that the material is closely confined to chambers, proportional counters and self-quenching Geiger counters, and that there is no discussion of crystal and scintillation counters or of electron multipliers. Further, the important work of Nawijn on permanent gas counters is omitted, and the very brief discussion of these counters contains a misleading and partly incorrect account of the quenching mechanism.

It is stated that the intention of the book is to provide the necessary theoretical background to enable new counters and chambers to be designed and used effectively. The material selected is treated in a scholarly manner, and the book will undoubtedly be of considerable assistance to serious users of these instruments. In addition, a great deal of practical information is included, and the book contains a good bibliography.

B. COLLINGE.

*Theorie der Supraleitung*. by M. von Laue. Pp. 114. 2nd Edition. (Berlin: Springer-Verlag, 1949.), No price.

The name of the author is so widely known in connection with x-ray crystallography that it is often not realized that he is also one of the pioneers in the development of our understanding of superconductivity. It was in fact he who first suggested a simple interpretation of the curious result found by de Haas in Leiden that the resistance of a superconducting wire was restored by a magnetic field at roughly half the field strength if the field was perpendicular instead of parallel to the wire. von Laue in 1932 pointed out that the lines of force of a magnetic field would have to flow round the outside of a perfect conductor and so, for a transverse wire, would have twice the strength at the 'equator' of the cross section as they would in the absence of the wire. This observation directed interest towards the general problem of the magnetic properties of superconductors, which in the early 1930's were intensively studied and proved to be as fundamental as the electrical properties.

The whole subject was considerably tidied up by the phenomenological theory of F. and H. London (published in 1935) which, while not explaining the origin of superconductivity, provided in a single equation a neat summary of all the then known electrical and magnetic properties. Moreover, a new parameter, the penetration depth of a magnetic field into a superconductor, was introduced which, although too small to show up in most experiments with macroscopic specimens, plays an important part in the behaviour of specimens of dimensions less than $10^{-4}$ or $10^{-5}$ cm. Experiments during the last ten years have confirmed the existence of this penetration depth and thus emphasized the importance of the Londons' theory as a first step towards a more fundamental understanding of superconductivity.

Von Laue took an active part in the early discussions concerning the correct formulation of the Londons' theory, and more recently has developed some aspects of its applications; the book under review is essentially an attempt at a systematic exposition of the Londons' theory, with particular emphasis on the author's own contributions. After a careful discussion of the fundamental assumptions, the consequences of applying the theory to problems such as the distribution of current in a superconducting wire and the distribution of magnetic field round superconductors of various shapes are presented in full mathematical detail. Although for most practical purposes only the limiting forms of the solutions for very large or very small dimensions are required, it is useful for the research worker to have the general solutions collected together in one place, together with a thorough discussion of the fundamental assumptions made.

In the later parts of the book other problems are discussed in which special assumptions have to be made in addition to the assumptions already implicit in the Londons' theory. In at least one instance, that of the resistance of a superconductor to high-frequency currents, it is unfortunate that the choice of special assumptions is one which has been shown to be unsuitable. The whole analysis of this problem given in this book is in fact not applicable, for as Pippard, in particular, has shown, the fact that the electronic mean free path becomes greater than the classical skin depth completely modifies the nature of the skin effect. It is a pity that in preparing this second edition the author has not taken the opportunity of bringing his treatment of this problem up to date. A similar criticism may be levelled too at the discussion of experimental data given at various places in the book. In the first edition, due presumably to the difficulties of consulting literature in the period just after the war, much of the experimental work quoted had already been superseded, or even proved wrong, by more recent experiments. It is disappointing to find that in this respect practically no changes have been made in the second edition, apart from occasional references to newer work in footnotes. The main novelty of the second edition compared with the first is the introduction of generalizations of the theory to take account of possible anisotropic effects in non-cubic superconductors and also of possible non-linear effects. The experimental evidence on both these questions is still very meagre, so that these generalizations seem a little academic, but they may prove their value in the future by suggesting new experiments and providing the basis for their interpretation.

It will be seen that while the book will undoubtedly prove of value and interest to the specialist, it cannot be recommended unreservedly as a well-balanced survey of the present state of the subject. D. S.

*Voltage Stabilizers*. By F. A. BENSON. Pp. 125. 1st Edition. (London: Electronic Engineering Press, 1950.) 12*s*. 6*d*.

The literature on the subject of voltage stabilization is now very extensive, and much of it not readily accessible. This small book will therefore serve as a valuable guide and source of reference to the subject. It is divided into four sections dealing respectively with the systems employing magnetically saturated elements, glow-discharge tubes and thermionic valves and with miscellaneous other methods. The discussion of the merits and limitations of the alternative arrangements is supported by mathematical arguments which will be readily followed by most readers. The book is well presented and indexed, and contains an extensive bibliography. E. H. T. JACKSON.

*Electrical Communication*, by A. L. ALBERT. Pp. ix + 593. 3rd Edition. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, 1950.) 52*s*.

According to the author this book has been designed as a college text to cover the entire field of electrical communication. If, however, one excludes the chapters on filters and transmission lines it is a descriptive account of the field. It contains no statement of Kirchhoff's equations or of the mesh and nodal systems of analysis and no discussion of circuit transients; the sections of the book devoted to valve circuits and radio-frequency systems are outlines only. It would have been better if the author had provided a more thorough grounding in circuit analysis and restricted the scope of the book to line communication.

The first chapter is a brief history of electrical communication prior to 1936. Chapter 2 is called " Fundamentals of Acoustics " but is a readable survey of reverberation, ear physiology, and the characteristics of speech and music. Circuit theory is introduced in chapter 3 without mentioning Kirchhoff's equations or the mesh and nodal systems of analysis, whilst chapter 4 contains a good description treatment of microphones, loudspeakers and telephone receivers. Chapters 5, 6 and 7 discuss filter circuits, transmission lines and cables respectively at degree course level. Chapter 8 is called " Electronic Applications of Communication ". It commences with valves and their characteristics and covers eight types of amplifier (including the travelling wave variety), eight types of oscillator (including the magnetron and klystron), frequency multiplication, neutralizing, negative feedback, the cathode-ray oscilloscope and semiconductors; this amount of material is treated in 41 pages. The next three chapters on telegraph and telephone systems may be useful to students taking the examination of the City and Guilds of London Institute in these subjects. Chapters 12 and 13 are concerned with radio wave propagation and radio systems; they do not meet the needs of students who intend to specialize in light current electrical engineering. The last chapter is a survey of interference between power systems and telephone lines.

G. R. BROOKS.

*The Inelastic Behaviour of Engineering Materials and Structures*, by A. M. FREUDENTHAL. Pp. xvi+587. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, Ltd., 1950.) 60*s*.

Those who have to design engineering structures are always faced with the problem of judging the mean between flimsiness and massiveness in the components they use. The occasional failure of large static structures such as bridges shows that even here, where undue massiveness can most readily be tolerated, the problem is not an easy one; it is vastly more difficult in the design of engines and aircraft where lightness is so important. Due to recent engineering developments, the need to use the materials of construction as effectively as possible has become even more intense. In gas turbines, for example, the attainment of the highest possible working temperatures is essential; even if the components no longer behave purely elastically, and creep takes place, at these temperatures this has to be tolerated, and one has to try to design on the basis of a limited working life.

Professor Freudenthal considers that, in order to obtain the maximum service from engineering components and at the same time to control the risk of failure in them, the engineer must pay more attention to their inelastic properties. He also considers that it is necessary to have a sound understanding of the physics of plastic deformation before one can apply the mathematical theories of plasticity intelligently to problems of design. To meet these needs he has attempted, in the present book, to cover the whole subject of inelastic deformation, dealing with all aspects from its origin in the atomic and crystalline structures of materials to its significance in the theory of the design of structures.

The book covers an enormous range of subjects. It starts with introductory chapters dealing with atomic physics and the structure of matter, and goes on to discuss from this point of view the processes of slip, work hardening, thermal softening, creep, fatigue and fracture. An account is given of the rheological properties of non-crystalline substances. Formal theories of inelastic deformation are discussed at great length; such topics as the mechanical equation of state, ideal rheological bodies and their properties, and relations between creep, stress relaxation and internal friction are included. Applications of these theories are made to problems of torsion, bending and the deformation of tubes under pressure, to the technological processes of pressing, rolling and drawing, and to the general problem of structural design. The final chapter discusses the significance of the results of mechanical tests.

One cannot but admire the energy which has gone into the assembling of information on so many diverse topics and the collecting of numerous up-to-date references to publications on each of them, and the skill with which they have been dovetailed together to give a coherent account of the whole field. On the other hand, the clarity of exposition is not as good as it could be. This is partly due, one feels, to the author being not quite sure of himself when discussing subjects, such as the physical aspects of plastic deformation, which lie on the fringe of his own interests. Even at places where sharp, clear definitions are possible, e.g. when introducing the notions of stress (p. 171) and strain (p. 177), these definitions are obscured by a flood of words. Because of this it is doubtful whether the book will be successful as an introductory textbook for engineers, for whom it was written. Rather, its most useful function will be to enable those who are already expert in one branch of the field to familiarize themselves with what is going on in other branches. This is sufficient justification for the book.

A. H. C.

### *The Response of Physical Systems*, by J. D. Trimmer. Pp. ix + 268. (New York: John Wiley and Sons, Inc.; London: Chapman and Hall, 1950.) 40*s*.

The particular merit of this book is that it draws attention to the generality of its subject, the response of 'physical systems' to applied forcing stimuli; numerous worked examples are included, drawn from electrical and mechanical circuits, chemical, thermal and nuclear chain reactor systems. Such generality has, of course, become almost a fashion nowadays; a system of applied mathematics initiated mainly by Rayleigh and Kelvin, and later greatly developed, largely by electrical engineers, is being discovered and put to use by engineers from many other fields—one might add, not before this was due.

The approach adopted in the book is through the classical solution of differential equations, rather than by operational means. For a book of this particular level such a choice is perhaps wise, since the physical principles are better kept in view. An appendix gives some introduction to Laplace transforms, unfortunately not interpreted through the complex variable but rather as a collection of the essential formulae.

This is a general criticism; although the book is broad in scope, the bulk of the material is given superficial treatment reduced, in many cases, to the mere statement of formulae, with little introduction or derivation from fundamental concepts. In particular this applies to the treatment of Fourier analysis, of natural modes of oscillation, of the 'gain function' of a system, and of the wave equation. The section on feedback systems is elementary and contains little of modern theory. Again, the treatment of the 'inverse problem' of obtaining a system to possess a required behaviour begs the whole difficult question of circuit synthesis. In view of this it is difficult to say for which type of reader the book would be intended. Probably it would be of most interest to college students of mathematics who wish to be introduced to engineering applications of differential equations; it would at least supply them with many of the definitions and concepts used.

In its favour this book has some treatment of systems with variable parameters, of non-linearity and also of simple statistics, aspects which are not usually included in elementary texts on circuit theory.

COLIN CHERRY.

### *Wave Theory of Aberrations*, by H. H. Hopkins. Pp. viii + 169. 1st Edition. (Oxford: Clarendon Press, 1950.) 15*s*.

Consideration of the phase at which a disturbance in optical imagery arrives at the focus is of great importance and, indeed, completes the information obtained by ray tracing. Optical designers ignored this aspect till its significance was pointed out by Chalmers and Conrady, whose work is not yet fully appreciated, so that this book,, which makes use of the form of the wave front to deduce the aberrations, will be welcomed.

The form of the wave front is derived from the results of trigonometrical ray tracing and the phase from the optical path differences. The author also develops the first-order aberrations in terms of the data of a system of centred spherical surfaces, and extends the treatment to aspheric surfaces. The scheme of computation is shown.

The value of the wave treatment in the assessment of the complete aberrations of a system is undoubted; what is not so apparent is its value in the designing of optical systems. The author's first-order aberrations are derived from the paraxial angles, which can only be found if some sort of system is assumed to start with. What a student wants to know is how to arrive at the initial form. Once he has got this it does not matter much what system of computation is used. The experienced designer will welcome a fresh *modus operandi* if it offers any advantage either in speed or insight. In this book the author has completely revealed a method which is open to anyone to try for himself.

The notation used is that of Conrady, probably a wise choice, as this is becoming standard in England and the U.S.A. The disadvantages of this notation are well known, and in the book are apparent in the rather clumsy appearance of the formulae—which is not the fault of the printer. There are few misprints and these not serious, and happily few lapses such as that appearing on page 135: "In dealing with a thin lens . . . . the spherical aberration . . . . may be easily calculated, . . . . using the expression . . . ."

The book is easy to read and demands no mathematical background beyond simple calculus. The author is to be congratulated on his clear exposition. It should be widely read.

H. W. L.

*Der Ultraschall* (*und seine Anwendung in Wissenschaft und Technik*), by L. BERGMANN. Pp. 748. 5th Edition. (Zurich: Hirzel Verlag, 1949.) 50 fr.

This is the fifth edition of Dr. Bergmann's book and the first to appear since the end of the war. The first edition was published in Germany in 1937 and an English edition (London: Bell and Sons) in 1938. The second German edition appeared in 1939, the third in 1943 and the fourth in 1944. The fourth edition is extremely rare as it was almost entirely destroyed in the fighting for Berlin. In spite of its continuous revision, the very rapid publication of new knowledge since the war has required the complete rewriting and considerable enlargement of the text for this edition, which is much more up to date than its most recent predecessor.

Dr. Bergmann covers the whole subject systematically with extraordinary completeness. The book is divided into two main parts, the first, containing 230 pages, describes the production, detection and measurement of ultrasonic waves, and the second, what are broadly described as applications. The first chapter describes various types of ultrasonic generator and includes a very complete discussion of the piezoelectric effect in quartz and the use of a quartz crystal as a transmitter and receiver. The second chapter, devoted to detection and measurement of the waves, starts with a formal treatment of reflection at, and transmission through a boundary, goes on to discuss mechanical and thermal methods of investigating the sound field and the construction and theory of the ultrasonic interferometer, and concludes with a particularly full account (70 pages) of the many elegant optical procedures for measuring and demonstrating the properties of the ultrasonic field. This section is particularly well illustrated by both photographs and diagrams, as we should expect since this part of the subject is one to which Dr. Bergmann has himself so largely contributed.

In the second main part of the book Dr. Bergmann first reviews what is known concerning the velocity and absorption of sound in solids, liquids and gases, including the techniques of measurement and the various theoretical treatments. This section of the book is specially valuable for a series of tables in which the values of the velocity and the coefficients of absorption for a great variety of substances are quoted. Where, as in the case of absorption measurements, there is disagreement between different sources, the author has brought all the values together to allow comparison to be made. The review of the determination of the elastic constants of crystals brings together a great deal of new material which is very thoroughly described. The author then goes on to describe the many technological applications using ultrasonic methods including television systems, depth sounding, applications to radar and in communication practice, material testing, precipitation and preparation of colloids and emulsions, etc. Medical and biological effects are also adequately described. These last-mentioned sections are naturally more descriptive than the rest of the book but the author has used a critical approach in reporting published work.

The photographs and diagrams, which number 460, are very clear and excellently reproduced and the book is well printed and bound.

A most valuable feature is the bibliography of over 2,300 items covering the period up to early 1949, which must be by far the most complete ever published. It is arranged in alphabetical order of authors' names, and there are very many references to it in the text. There are also complete name and subject indexes to the text itself.

The author is to be warmly congratulated on the production of a work of such scholarly completeness, which will be welcomed by all those interested in any of the many subjects discussed. J. M. M. P.

*Piezoelectric Crystals and their Applications to Ultrasonics*, by W. P. MASON. Pp. xi + 508. (London: Macmillan; New York: D. Van Nostrand Co., Inc., 1950.) 56*s*.

The physicist or electrical engineer who works with piezoelectric crystals is indeed fortunate in having two such monumental treatises (500 pages each) on the subject as those of Cady and the present author, appearing at about the same time. The subject has made such rapid advances into several branches of physics that one tends to forget that it is only seventy years since the discovery of the effect by the brothers Curie and that Jacques lived on into the period of the recent Vichy government in France.

Dr. Mason is in charge of a group working on this subject at the Bell Telephone Laboratories, through whose auspices the book is issued. He is distinguished for his discoveries of ferro-electric crystals and their use as transducers for ultrasonic work, particularly in eliciting shear vibrations in solids and liquids.

This work is quite naturally given prominence in this book, but it is more than an exposition of the author's own researches. The whole domain of piezoelectricity as far as it has been explored is covered in his pages. Though detailed theories of the piezoelectric effect and its modern developments to complex crystals like Rochelle salt, the dihydrogen phosphates (' ferro-electrics ') and the electrostrictive effect in ' barium titanate ' are included, the emphasis throughout is on practical applications.

The final chapters deal with the behaviour of these systems as transducers and their applications to measurements in liquids and solids.

Althogether this is an excellent work which makes available much information to those concerned in piezoelectricity and its applications which has not been hitherto available.

E. G. R.

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# ABSTRACTS FOR SECTION A

## *Thermal Conductivity of Dense Matter*, by K. S. Singwi and M. K. Sundaresan.

*ABSTRACT.* The thermal conductivity of degenerate matter is calculated taking account of the effect of relativistic mechanics. The result is put in a form suitable for astrophysical calculations. The two limiting cases of non-relativistic degeneracy and extreme relativistic degeneracy are discussed.

## *The Saturation Magneto-Resistance of Ferromagnetic Alloys*, by R. Parker.

*ABSTRACT.* It is suggested that the magneto-resistance coefficient of a ferromagnetic alloy is composed of two terms, one associated with the temperature-dependent, the other with the temperature-independent contribution to the electrical resistivity. An equation based on this hypothesis is in good agreement with the experimental results for silicon–iron alloys. The validity of the equation for other alloys is discussed.

## *The Reaction $^7Li(d,\alpha)$ and the Ground State of $^5He$*, by A. P. French and P. B. Treacy.

*ABSTRACT.* A differential ionization chamber with electron collection has been designed for the study of α-particle spectra. Two chambers of this type have been used to investigate α–α coincidences in the reaction $^7\mathrm{Li}(\mathrm{d}, \alpha \mathrm{n})^4\mathrm{He}$. The results confirm that this process takes place, and the observed angular correlation of successive α-particles lends some support to the view that the virtual ground state of $^5\mathrm{He}$ is $\mathrm{P}_{3/2}$.

## *On the Experimental Determination of the Lifetimes of Atomic Energy States*, by G. Stephenson.

*ABSTRACT.* The lifetimes of certain atomic resonance states have been measured using a magnetic rotation method devised by Weingeroff in 1931. This method possesses the advantage of not requiring an accurate knowledge of the vapour pressure data. The experimental results are compared with theoretical values obtained using the recent Bates–Damgaard Coulomb approximation, and with results based on Hartree and Slater type wave functions. Good agreement is found with the Bates–Damgaard results.

## *Precise Determination of Lattice Constants by Electron Diffraction and Variations in the Lattice Constants of Very Small Crystallites*, by F. W. C. Boswell.

*ABSTRACT.* Previous workers have reported that lattice constants determined by electron diffraction differ from the accepted x-ray values. It has been suggested that these differences may be due to a change in lattice constant with crystal size. To examine this question further, the lattice constants of several alkali halides have been determined, by electron diffraction, to a higher accuracy than had previously been attained. The x-ray value of the lattice constant of TlCl was used as a standard reference. The lattice constants of six alkali halides, with crystal sizes greater than 150 A., were found to agree with the x-ray values to within the experimental error of 0·05%. It has been shown that for crystal sizes less than 100 A. the lattice constants of the alkali halides were found to be less than the x-ray values by an amount dependent on crystal size. For a crystal size of about 30 A. the decrease was approximately 0·5%.

Similar determinations on evaporated gold films showed that, for a crystal size of about 20 A., the decrease in the lattice constant was almost 2%. For a crystal size of 40 A., the largest obtained, the lattice constant was 0·2% less than the x-ray value. Evaporated films of silver and bismuth also were found to have lattice constants less than the x-ray values.

It can therefore be concluded that previous reports of an increase of the lattice constants of alkali halides with decreasing crystal sizes are in error. Some of the discrepancies noted by previous workers may have resulted from assigning the x-ray value to the lattice spacing of gold films or gold foil used as a standard reference.

### *The Parameters of Simple Excess Semiconductors,* by P. T. Landsberg, R. W. Mackay and A. D. McRonald.

*ABSTRACT.* Graphical information is given which facilitates the evaluation of the parameters entering into the theory of simple excess semiconductors.

### *The Band-Spectrum of Carbon Monofluoride, CF,* by E. B. Andrews and R. F. Barrow.

*ABSTRACT.* Measurements of two band-systems attributed to CF are reported, A rotational analysis of four bands, 1,0, 0,0, 0,1 and 1,1, of a transition A $^2\Sigma^+$—X $^2\Pi$ is given and the vibrational analysis of a system B (near case–b)—X $^2\Pi$. The rotational constants of the B state have been estimated from band-head separations. X $^2\Pi$ is probably the ground-state of CF. The initial covergence of the vibrational levels in B is rather rapid, and suggests that $D_0''$ is about $4{\cdot}9_6$ ev. Constants derived are as follows :

| State | $\nu_e$ | $\omega_e$ | $x_e\omega_e$ | $B_e$ | $\alpha_e$ | $r_e$ (A.) |
|---|---|---|---|---|---|---|
| B | $49451{\cdot}_8$ | $1191{\cdot}_0$ | $19{\cdot}_4$* | $1{\cdot}32_0$ | 0·025 | $1{\cdot}31_8$ |
| A $^2\Sigma^+$ | 42705 | 1764 | $2_2$ | $1{\cdot}727_7$ | $0{\cdot}026_0$ | $1{\cdot}151_6$ |
| X $^2\Pi$ · | 77<br>0 | $1308{\cdot}_4$ | $10{\cdot}8_6$ | $1{\cdot}419_0$ | $0{\cdot}019_0$ | $1{\cdot}270_8$ |

* $-0{\cdot}_4(v+\frac{1}{2})^3$

### *Investigation of the γ-Rays from Polonium,* by M. A. Grace, R. A. Allen, D. West, and H. Halban.

*ABSTRACT.* A method of standardizing polonium sources by measurement of the number of $\gamma$-rays is described. The hard $\gamma$-radiation of 0·773 Mev. energy emitted in the decay of $^{210}$Po has been found to have an intensity of $1{\cdot}8 \pm 0{\cdot}14 \times 10^{-5}$ quanta per $\alpha$-particle. The internal conversion of this line to the extent of $6{\cdot}7 \pm 1{\cdot}7\%$ gives rise to K X-radiation of lead. No other soft radiation was detected. The decay scheme of polonium is discussed.

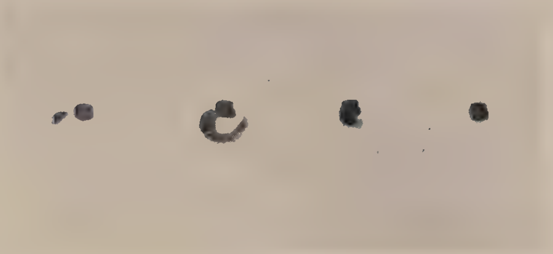

Figure 3. Sequence of setting photographs. The alignment of the capillary improves from left to right. For large misalignments the halo is incomplete. On aligning, the halo becomes complete and shrinks, until finally only a central spot remains, an approximate image of the focus. The spot on the right indicates correct alignment.

Figure 5 (*a*). Back reflection photograph of cold-worked aluminium (422 ring) examined with normal type of back reflection camera. Beam diameter $\sim 1$ mm. at specimen.

Figure 5 (*b*). Back reflection photograph of cold-worked aluminium (422 ring) examined with a micro-beam of diameter $\sim 35\,\mu$ at the specimen.

Figure 10. Variation of focal pattern from line filament system; bias increasing from left to right.

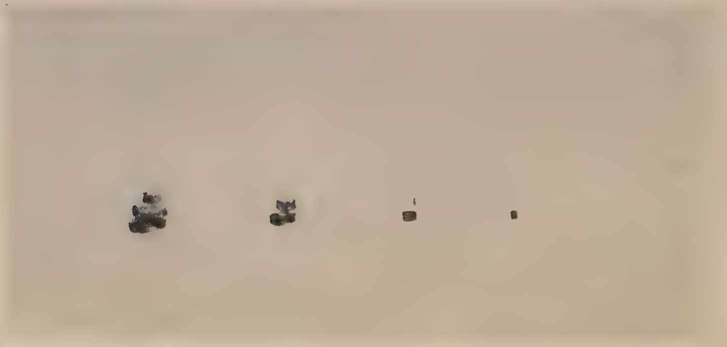

Figure 12. Variation of focal pattern from hairpin filament; bias increasing from left to right.

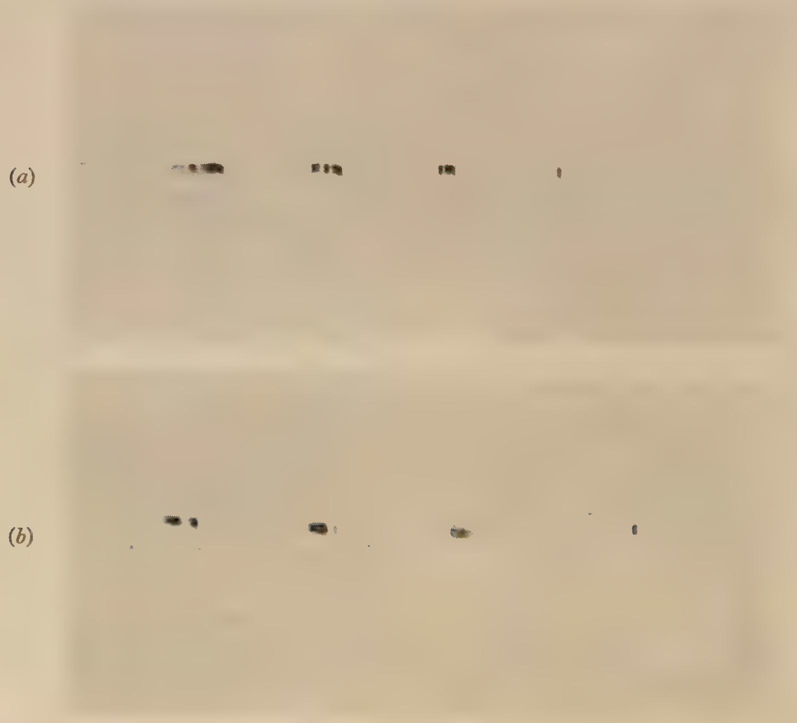

Figure 14. Variation of focal patterns from strip filaments; bias increasing from left to right. (Due to asymmetry, one outer band of the zero bias patterns is of weaker intensity than the other; both bands are visible on the original film, but one does not show in the reproduction.)

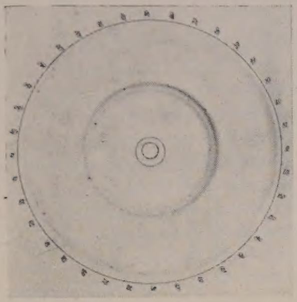

Figure 1. ½ hr.

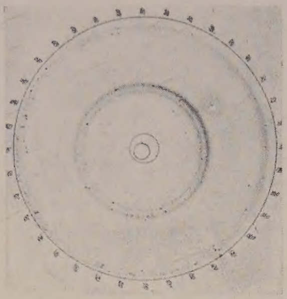

Figure 2. 4½ hr.

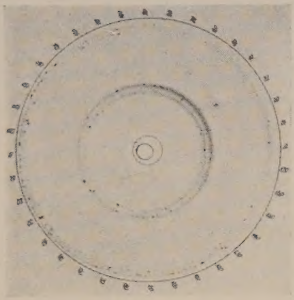

Figure 3. 8½ hr.

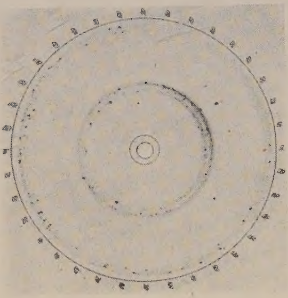

Figure 4. 48½ hr.

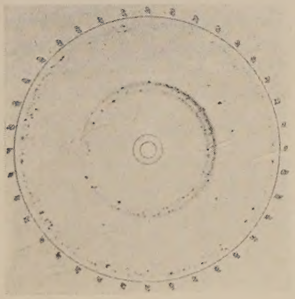

Figure 5. 75 hr.

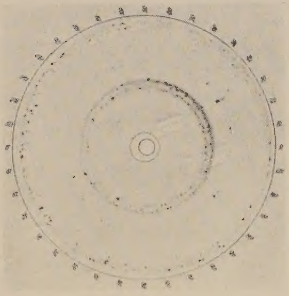

Figure 6. 172 hr.

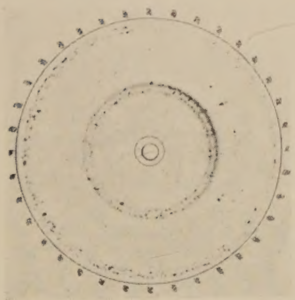

Figure 7. 273 hr.

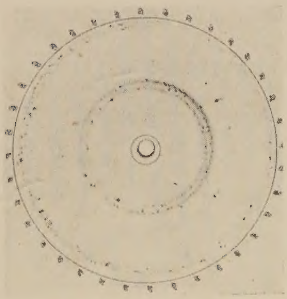

Figure 8. 337 hr.

Back-reflection x-ray photographs taken with a specimen compressed 62%, at different times after the release of stress, the radiation falling normally on the surface of the specimen.

Plate I.

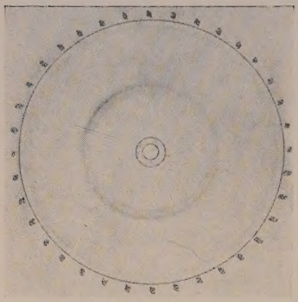

Figure 10. ¾hr. (0°).

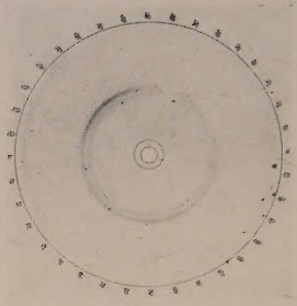

Figure 11. 357 hr. (0°).

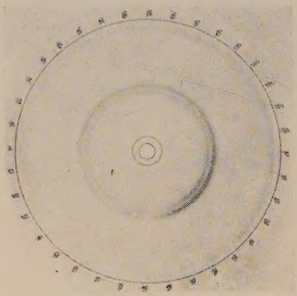

Figure 12. 2¾ hr. (20°)

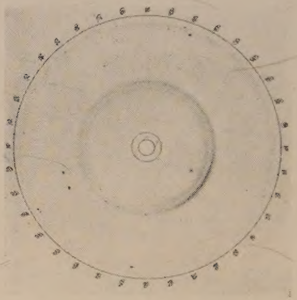

Figure 13. 380 hr. (20°).

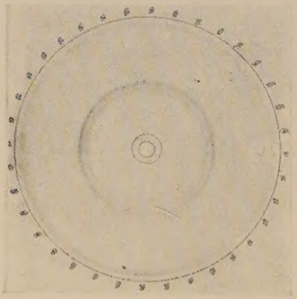

Figure 14. 5 hr. (30°).

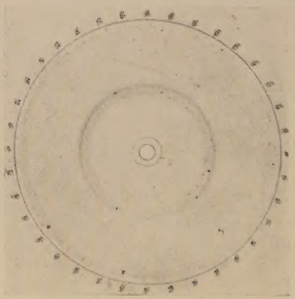

Figure 15. 359 hr. (30°).

Back-reflection X-ray photographs taken with a specimen compressed 56·5%, at different times after the release of stress, the radiation falling on the surface at different angles of incidence. (Cobalt radiation; (420) planes.)

PLATE II.

# THE PHYSICAL SOCIETY

## MEMBERSHIP

Membership of the Society is open to all who are interested in Physics:

FELLOWSHIP. A candidate for election to Fellowship must as a rule be recommended by three Fellows, to two of whom he is known personally. Fellows may attend all meetings of the Society, are entitled to receive Publications 1 (either Section A or Section B), 4 and 5 below, and may obtain the other publications at much reduced rates.

STUDENT MEMBERSHIP. A candidate for election to Student Membership must be between 18 and 26 years of age and must be recommended from personal knowledge by a Fellow. Student Members may attend all meetings of the Society, are entitled to receive Publications 1 (either Section A or Section B) and 4, and may obtain the other publications at much reduced rates.

Books and periodicals may be read in the Society's Library, and a limited number of books may be borrowed by Fellows and Student Members on application to the Honorary Librarian.

Fellows and Student Members may become members of the *Colour Group*, the *Optical Group*, the *Low Temperature Group* and the *Acoustics Group* (specialist Groups formed in the Society) without payment of additional annual subscription.

## PUBLICATIONS

1. *The Proceedings of the Physical Society*, published monthly in two Sections, contains original papers, lectures by specialists, reports of discussions and of demonstrations, and book reviews. Section A contains papers mainly on atomic and sub-atomic subjects; Section B contains papers on macroscopic physics.

2. *Reports on Progress in Physics*, published annually, is a comprehensive review by qualified physicists.

3. *The Handbook of the Physical Society's Annual Exhibition of Scientific Instruments and Apparatus*. This Exhibition is recognized as the most important function of its kind, and the Handbook is a valuable book of reference.

4. *The Bulletin*, issued at frequent intervals during the session, informs members of programmes of future meetings and of the business of the Society generally.

5. *Physics Abstracts* (*Science Abstracts A*), published monthly in association with the Institution of Electrical Engineers, covers the whole field of contemporary physical research.

6. *Electrical Engineering Abstracts* (*Science Abstracts B*), published monthly in association with the Institution of Electrical Engineers, covers the whole field of contemporary research in electrical engineering.

7. *Special Publications*, critical monographs and reports on special subjects prepared by experts or committees, are issued from time to time.

## MEETINGS

At approximately monthly intervals throughout each annual session, meetings are held for the reading and discussion of papers, for lectures, and for experimental demonstrations. Special lectures include: the *Guthrie Lecture*, in memory of the founder of the Society, given annually by a physicist of international reputation; the *Thomas Young Oration*, given biennially on an optical subject; the *Charles Chree Address*, given biennially on Geomagnetism, Atmospheric Electricity, or a cognate subject; and the biennial *Rutherford Memorial Lecture*. Meetings are generally held each year at provincial centres, and from time to time meetings are arranged jointly with other Societies for the discussion of subjects of common interest.

Each of the four specialist Groups holds about five meetings in each session.

## SUBSCRIPTIONS

Fellows pay an Entrance Fee of £1 1*s*. and an Annual Subscription of £3 3*s*. Student Members pay only an Annual Subscription of 15*s*. Second Section of *Proceedings* 30*s*. No entrance fee is payable by a Student Member on transfer to Fellowship.

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